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ABSTRACT

This volume contains the papers and comments from the Workshop on Mathematics Education, a special session of the 15th Conference on Stochastic Processes and Their Applications, held in Nagoya, Japan, July 2-5, 1985. Topics covered include: (1) probability; (2) statistics; (3) deviation; (4) Japanese mathematics curriculum; (5) statistical education; (6) programmable calculators; (7) limit theorems; (8) Poisson distribution; (9) French curriculum for high school and college; (10) middle school curriculum, People's Republic of China; (11) New Zealand curriculum; and (12) reforms in Korean curriculum. (JM)

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Mathematics Education

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Stochastic Processes and their Applications

Fifteenth Conference

July 1985

Nagoya, Japan

FOREWORD

The Workshop on Mathematics Education was held on July 3 and 4, 1985, as a special session of the Fifteenth Conference on Stochastic Processes and their Applications which was held in Nagoya, Japan, for the period July 2 – 5, 1985. The Workshop was attended by more than hundred mathematicians including university professors from all over the world as well as local high school teachers.

The papers and comments in this volume are arranged in the order of their presentation at the session. To our regret, the record of the discussion after each talk can not be included here.

The editors are grateful to all the speakers who delivered very interesting lectures at the session and have written the papers based on their lectures for publication in this volume.

January, 1986

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PROBABILITY AND STATISTICS IN THE PRESENT AND DESIRED MATHEMATICS CURRICULA FOR JAPANESE HIGH SCHOOLS

Hiroshi FUJITA

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In this paper we want to review the present state and analyze current problems of mathematics education in Japanese junior and senior high schools, paying a particular attention to teaching of probability and statistics. We also wish to express our proposals towards reformation of the whole and the corresponding part of the mathematics curriculum.

1. Japanese school system and the national curriculum

We begin with a review of the Japanese school system which is quite similar to the US system. It is remarkable that the curricula of all subjects for Japanese schools, private schools being included, up to senior high schools are set by the Ministry of Education through the Course of Study, which is a kind of national curriculum. The Course of Study is usually revised every ten years. The present one was set in 1977 for junior high schools and in 1978 for senior high schools.

The present curriculum is in nature a kind of post-modernization curriculum.

2. Probability and statistics in the national curriculum

Although some primitive consideration of statistical data and combinatorial analysis appears even at the 6th grade (the last year of the elementary school), the concepts of probability and arrangement of statistical data are first taught at the lower secondary education (education at junior high schools, the 7th grade – the 9th grade), which is a compulsory education.

The upper secondary education (at senior high schools) is not compulsory. However, in recent years 94% of the age cohort proceed to senior high schools, where mathematics is a compulsory subject, Mathematics I, in the first year but is optional for the following years. Incidentally, mathematics up to the 10th grade is organized in the form of an integrated curriculum, while mathematics for the 11th and 12th grades is divided into separated courses; Algebra and Geometry, Basic Analysis, Probability and Statistics, Differential and Integral Calculus. Mathematics II is an exception and is composed of topics extracted from other optional subjects in a softened version. Thus, topics on probability and statistics are only taught in the subject, Probability and Statistics, together with combinatorial mathematics. This point is different from the case of the preceding national curriculum where concepts of probability and basic laws of probability as well as the law of the conditional probability were included in the compulsory Mathematics I. An idea of the status of probability and statistics in

the present national curriculum could be gained through the Appendices (A) and (B) which present a sketch of the whole mathematics curricula for junior high schools and senior high schools, respectively.

3. General surroundings and specific difficulties

In order to identify difficulties in teaching mathematics, particularly, those in teaching probability and statistics in Japanese high schools, it may be appropriate to start with a review of the following social and global factors:

- a) Positive attitude of the society towards higher education and recognition of importance of mathematics as a desired competence of intellectual citizens;
- b) Saturating high ratio of advancement to high schools;
- c) Over-heated competition at the entrance examinations;
- d) Qualified but conservative teaching forces.

As specific difficulties in teaching probability and statistics are often mentioned the following:

- i) Topics are too fragmental and the curriculum is too crowded;
- ii) Probability is now accepted as an established field of high school mathematics. On the contrary, students are not sufficiently mature mentally nor well-prepared technically, in order to learn statistics as a topic of high school mathematics;
- iii) Influences or distortions by the entrance examinations are serious.

To be fair, we should mention also the positive aspect of influences of the entrance examination, which encourage intensified learning through and for the problem-solving activities. In Appendix (C), we give English translation of some samples of problems of the entrance examinations, which show the level of the mathematics problems concerning probability at the entrance examinations.

4. Some proposals

Below we refer to some points of our proposals for reformation or improvement of our mathematics curriculum, which came out of investigations and discussions of our study group on mathematics education.

- a) Fundamental policy: In countries like Japan the central goal of mathematics education at the secondary level should be cultivation of mathematical intelligence.
- b) Objectives: Mathematics education should provide the majority of students with "the mathematics literacy" and provide the brighter students with a deep potential which will support their academic and professional progress.
- c) Strategies: i) The national curriculum should be of a flexible structure, for instance, of the core-and-options type. ii) Inspiring problems should be introduced in the text books. iii) Use of calculators should be encouraged in order to enrich students' mathematical experiences. iv) In order to provide the students with more chances of personalized learning with more vivid motivations and with more convincing mathematical experiences, school use of personal computers should be developed.

The author believes that statistics should be taught around in the 9th grade as a part of

mathematics literacy, while, for example, the idea of the testing of hypothesis may be mentioned only as an example of application of probability distribution at the 11th or 12th grades. Thus, mathematics in the senior secondary education should emphasize probability.

Use of calculators will facilitate teaching of some transcendental functions like $\exp(-\alpha x^2)$ at an early stage, which must reduce the difficulty in teaching normal distributions. Simplest numerical quadrature of these functions by means of calculators will pave the way of teaching normal distributions without the knowledge of proper or improper integrals.

Finally, classroom use of personal computers could give better motivations and better understanding of concepts in learning stochastic topics through various simulations. Fortunately, a strong movement supported by the Ministry of Education is going to take place to provide all schools with a number of personal computers and to introduce active uses of computers for school teaching.

APPENDIX (A)

CURRICULUM OF MATHEMATICS FOR THE LOWER
SECONDARY SCHOOL IN JAPAN

revised by the Ministry of Education in 1977,
and put in force from 1981.

Incomplete excerpt from the General Provisions of the Course
of Study for the Lower Secondary School

1. Mathematics is a required subject in each grade.
2. Standards for the total number of school hours in each grade are prescribed as follows:

7th grade (12 years old)	105
8th grade (13 years old)	140
9th grade (14 years old)	140

Notes: 1) A unit school hour is 50 minutes.

2) Lessons on Mathematics must be given over at least 35 weeks a year.

Mathematics

I. OBJECTIVES
II. OBJECTIVES AND CONTENTS IN EACH GRADE

[1st Grade]

Contents

A. Numbers and Algebraic Expressions

- (1) To help pupils deepen their understanding of the properties of integer.
 - a. Expression of the integer as a product of prime numbers.
 - b. Properties of divisor and multiple.
- (2) To help pupils understand the meaning of positive numbers and negative numbers, and be able to compute with those numbers.
- (3) To help pupils develop their abilities to express relations and rules in a formula through the use of letters, and to be able to calculate simple expressions.
- (4) To help pupils understand the meaning of an equation, and be able to solve a linear equation.
- (5) To help pupils understand approximate value and be able to use it properly under different circumstances.

(Terms - Symbols)

natural number, factor, the greatest common measure, the least common multiple, sign, absolute value, term, coefficient, similar term, $\leq$, $\geq$

B. Functions

- (1) To help pupils deepen their understanding of a function, through studying the relationship between phenomena, paying attention to the two variable quantities.
 - a. Change and correspondence.
 - b. Variable and domain.
- (2) To help pupils understand that a functional relation can be expressed by means of a table, graph, formula, etc., and be able to look into the characteristics of the relationship.
 - a. Meaning of coordinates.
 - b. To express a functional relation in a table, graph, formula, etc., and to look into the characteristics of change and correspondence by using them.

C. Geometrical Figures

- (1) To help pupils consider geometrical figures through various manipulation, and to deepen their understanding of figures in space.
- (2) To help pupils develop their abilities to construct figures that meets given conditions.
 - a. To construct basic figures such as the bisector of an angle, perpendicular bisector of a line segment, perpendicular etc.
 - b. To consider a figure as a set of points that meet certain conditions, and to construct the figure.
- (3) To help pupils develop their abilities to measure a figure.
 - a. Length of an arc, and area of a sector.
 - b. Surface area and volume of a cylinder, a conical solid and a sphere.

(Terms - Symbols)

body of revolution, arc, chord, π , //, $\perp$, $\angle$

[2nd Grade]

Contents

A. Numbers and Algebraic Expressions

- (1) To help pupils be able to make computations of a simple formula using letters, and the four fundamental operations.
- (2) To help pupils further develop their abilities to find the quantitative relationships in phenomena, and to express and apply such relationships in a formula by using letters.
- (3) To help pupils understand the meaning of inequality and be able to solve a linear inequality.
- (4) To help pupils understand the meaning of forming simultaneous equations or inequalities and their solutions, and thereby be able to solve them.
 - a. The meanings of linear equation with two variables and its solution.
 - b. To solve simple simultaneous linear equations or linear inequalities.

B. Functions

- (1)
 - a. Some phenomena may be described through the use of linear functions.
 - b. A linear equation with two variables may be considered to express the functional relations between two variables.
- (2) To help pupils understand the characteristics of linear functions and develop their abilities to use them.

- a. The equation of a straight line and the graph of a linear function.
- b. The ratio of change of a linear function is constant.

C. Geometrical Figures

- (1) a. Displacements by parallelism, symmetry, and rotation.
- b. The properties of parallel lines.
- c. The conditions of congruence for triangles.
- (2) To help pupils clarify the concepts of similarity of figures and to develop their abilities to consider the properties of figures by using the conditions of congruence or similarity of triangles.
- a. The meaning of similarity and the conditions of similarity for triangles.
- b. The properties of the ratio of segments parallel lines.
- c. Properties of triangles and parallelograms.

(Terms · Symbols)

opposite angle, interior angle, exterior angle, center of gravity, R , Δ , $\equiv$, ∞

D. Probability and Statistics

- (1) To help pupils collect data in accordance with their objectives, and to arrange this data by using tables and graphs, and thereby be able to ascertain the trends of the data.
- a. Meaning of frequency distribution and way of looking at histogram.
- b. Meaning of relative or cumulative frequency.
- c. Meaning of mean value and range.

(Terms · Symbols)

frequency, class

[3rd Grade]

Contents

A. Numbers and Algebraic Expressions

- (1) To help pupils understand the square root of the positive numbers, and to be able to use them.
- (2) To help pupils be able to expand and factor simple expressions.
- a. Multiplication of a simple linear expression.
- b. Expansion and factorization of an expression by use of the formulas indicated below:

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a + b)(a - b) = a^2 - b^2$$

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$
- (3) To help pupils understand quadratic equations.
- a. Quadratic equations and their solution.
- b. To solve quadratic equations by using factorization and the formulas for solution, etc.

(Terms · Symbols)

radical sign, rational number, irrational number, $\sqrt{\quad}$

B. Functions

- (1) a. Various phenomena and functions describing them.
 b. Functions which are in proportion to or in inverse proportion to x^2 .
 c. The ratio of change of functions.
- (2) With reference to two sets, to help pupils think about the relations of correspondence between their elements and deepen their understanding of the meaning of function.
 a. Sets and function.
 b. The domain and range of a function.

C. Geometrical Figures

- (1) To help pupils deepen their understanding of the properties of a circle.
 a. Circles and straight lines.
 b. Relationship of the angle of circumference to the central angle.
- (2) To help pupils understand the measuring properties of figures.
 a. The Pythagorean theorem and its application.
 b. Measurement of height and distance.
 c. The relationship between the ratios of length, area, and volume in similar figures.

(Terms - Symbols)

tangential line, point of tangency

D. Probability and Statistics

- (1) To help pupils understand about the probability by paying attention to the frequency obtained through large numbers of observations or trials.
 a. Meaning of probability.
 b. To compute probability in simple cases.
- (2) To help pupils understand that the trends in a population can be estimated from a sample.

Remarks concerning Contents

- (1) As for the content A-(3)-b, quadratic equations having the solution of real numbers should be taught. And, in using factorization as a solving method, the available formulas should be limited to the ones indicated in A-(2)-b.
- (2) As for the content D-(1), only events which can be easily classified by the aid of a tree diagrams, etc. should be dealt with.
- (3) As for the content D-(2), it should be limited to the level of dealing with experiments and observations.

APPENDIX (B)

CURRICULUM OF MATHEMATICS FOR THE UPPER
SECONDARY SCHOOL IN JAPAN

revised by the Ministry of Education in 1978,
and put in force from 1982.

A partial excerpt from the General Provisions
of the Course of Study for the Upper Secondary School

Part I. Mathematics Program for General Students

1. The subject "Mathematics" is composed of several subjects whose titles and associated credits are shown in the following table.

Subject	Standard Number of Credits
Mathematics I	4
Mathematics II	3
Algebra · Geometry	3
Basic Analysis	3
Differential and Integral Calculus	3
Probability · Statistics	3

(1 credit is 35 lesson-hours of 35 fifty-minutes periods of study throughout the year.)

I. *Mathematics I* (Compulsory; No topics on probability and statistics)

1. Objectives
2. Contents
 - (1) Numbers and Algebraic Expressions
 - a. Numbers
 - (i) numbers and sets
 - (ii) integers, rational numbers, real numbers
 - b. Algebraic Expressions
 - (i) polynomial
 - (ii) rational expressions
 - (2) Equations and Inequalities
 - a. Equations
 - (i) quadratic equations
 - (ii) simple equations of higher degree
 - (iii) simultaneous equations
 - b. Quadratic Inequalities

- c. Algebraic Expressions and Proof
(Terms · Symbols)
discriminant, imaginary number, i , complex number
- (3) Functions
 - a. Quadratic Functions
 - b. Simple rational and irrational functions
(Terms · Symbols)
inverse function
- (4) Geometrical Figures
 - a. Trigonometric Ratios
 - (i) sine, cosine, tangent
 - (ii) sine theorem, cosine theorem
 - b. Plane Figures and Equations
 - (i) points and coordinates
 - (ii) equations of straight lines
 - (iii) equations of circles
- (Terms · Symbols)
 $\sin$, $\cos$, $\tan$

II. Mathematics II (Optional; Topics on probability and statistics are included)

1. Objectives
2. Contents
 - (1) Probability and Statistics
 - a. Permutation, combination
 - b. Probability
 - c. Statistics
(Terms · Symbols)
 nPr , nCr , factorial, $n!$, complementary event, expectation, standard deviation
 - (2) Vectors
 - a. Vectors and their operations
 - b. Applications of vectors
 - (3) Differentiation and Integration
 - a. Meaning of differential coefficient
 - b. Derivatives and their applications
 - c. Meaning of integration
(Terms · Symbols)
limit value, $\lim$, indefinite integral, definite integral
 - (4) Sequences
 - a. Arithmetic sequence
 - b. Geometric sequence
 - (5) Various Functions
 - a. Exponential function
 - b. Logarithmic function
 - c. Trigonometric function

(Terms · Symbols)

power root, $\log_a x$, generalized angle

(6) Computer and Flow Charts

- a. Functions of electronic computers
- b. Algorithms and flow charts

3. Remarks concerning Contents

- (1) The content of Mathematics II (1)–(5) is basic to that treated in “III Algebra · Geometry”, “IV Basic Analysis” and “VI Probability Statistics”. The reader should refer to the “Contents” and “Remarks concerning Contents” for each of these subjects in order to gain a clearer understanding of the mathematical level of Mathematics II.
- (2) When actually teaching Mathematics II, the content of the class should be appropriately selected from among the above areas (1)–(6). In doing this, consideration should be given to the pupils’ ability and to the number of school hours assigned to the class.
- (3) In (6), actual experience preparing programs for the computer, running them, and analyzing the results should be included.

III. Algebra · Geometry (Optional; No topics on probability and statistics)

1. Objectives

2. Contents

(1) Conics

- a. Parabola
- b. Ellipse and hyperbola

(2) Vectors in a plane

- a. Vectors and their operations
- b. Inner product of vectors
- c. Applications of vectors
equations of straight lines and circles etc.

(3) Matrices

- a. Matrices and their operations
- b. Inverse matrix
- c. Linear transformation and mapping

(Terms · Symbols)

A^{-1}

(4) Space Figures

- a. Points, straight lines and planes in space
- b. Coordinates in space
- c. Vectors in space
include equations of straight lines, planes and spheres

IV. Basic Analysis (Optional; No topics on probability and statistics)

1. Objectives

2. Contents

(1) Sequences

- a. Simple sequences
 - arithmetic sequence, geometric sequence etc.
- b. Mathematical induction
- (Terms · Symbols) Σ
- (2) Functions
 - a. Exponential function
 - b. Logarithmic function
 - c. Trigonometric function
 - (i) generalized angle and circular measure method
 - (ii) trigonometric functions and their periodicity
 - (iii) addition theorems for trigonometric functions
- (Terms · Symbols)
 - power root, $\log_a x$
- (3) Variation of values of function
 - a. Meaning of differential coefficient
 - b. Derivatives and their applications
 - (i) derivatives of sums, differences and products multiplied by real number of functions
 - (ii) tangent, increase and decrease of values of function, velocity etc.
 - c. Integrations and their applications
 - indefinite integral, definite integral, area etc.
- (Terms · Symbols)
 - limit value, $\lim$

V. Differential and Integral Calculus (Optional; No topics on probability and statistics)

1. Objectives

2. Contents

(1) Limit

- a. Limit of a sequence
- b. Limit of value of a function

(Terms · Symbols)

convergence, divergence

(2) Differential Calculus and its Applications

a. Derivatives

- (i) differential calculus of product and quotient of functions
- (ii) differential calculus of composite and inverse functions
- (iii) derivatives of trigonometric functions
- (iv) derivatives of exponential and logarithmic functions

b. Applications of derivatives

tangent, increase and decrease of values of function, velocity, acceleration, etc.

(Terms · Symbols)

natural logarithm, e , the second derivative, point of inflection

(3) Integral Calculus and its Applications

a. Integral calculus

- (i) meaning of integration

- (ii) integrations by substitution and by part in simple cases
- (iii) integrations of various functions
- b. Applications of integration
 - (i) area, volume, length of path etc.
 - (ii) meaning of differential equations solving such equation as $\frac{dy}{dx} = ky$

VI. Probability · Statistics (Optional; Main topics are on probability and statistics)

1. Objectives

2. Contents

(1) Arrangement of Data

- a. Distribution of the variate
- b. Representative values and measures of dispersion

(Terms · Symbols)

variance, standard deviation, Σ

(2) Number of Cases of Possibilities

- a. Permutation combination
- b. Binomial theorem

(Terms · Symbols)

nPr , nCr , factorial, $n!$

(3) Probability

- a. Probability and its basic laws
- b. Independent trial and its probability
- c. Conditional probability

(Terms · Symbols)

complementary event, exclusiveness, independence, dependence

(4) Probability Distribution

- a. Random variable and its probability distribution
- b. Binomial distribution, normal distribution

(Terms · Symbols)

expectation

(5) Statistical Inference

- a. Population and sample
- b. Idea of statistical inference

(Terms · Symbols)

estimation, test

3. Remarks concerning Contents

Content (5) should be limited to gaining an understanding of statistical inference through concrete examples.

**THE CONSTRUCTION OF TEACHING PLANS AND REMARKS CONCERNING CONTENTS
IN EACH COURSE**

1. In designing teaching plans, the following should be considered.
 - (1) Mathematics I is required for all pupils in the 10th grade, but the other Mathematics Subjects are optional. Mathematics II, Algebra and Geometry, Basic Analysis and Probability and Statistics should be, as a rule, studied after Mathematics I.
 - (2) Differential and Integral Calculus should be, as a rule, studied after Basic Analysis.
2. In teaching, the following should be considered.
 - (1) When pupils study subjects beyond Mathematics I, the teacher must take into account the relation between the content of the various subjects, and also the systematization of the curriculum.
 - (2) (Terms · Symbols) written in each subject above are included to clarify the extent and range which are treated in the subject. In teaching, these terms and symbols should be closely related to the content of the subject.
 - (3) Wherever possible pupils should be encouraged to use computers or other mechanical aids to computation.

APPENDIX (C)

EXAMPLES OF MATHEMATICS PROBLEMS IN PROBABILITY
(AND STATISTICS) AT THE ENTRANCE EXAMINATIONS
TO JAPANESE NATIONAL UNIVERSITIES

I. 1985 CPEE (Common Primary Entrance Examination to National and Public Universities)

[1] Let P move along the periphery of a regular pentagon $ABCDE$, starting from the vertex A . Suppose that at exactly 1 second after P arrives at any vertex, it will transit to either one of the neighboring vertices, the transition probability being $\frac{1}{2}$ for each direction respectively. Then

- (i) The probability that P is located at E 3 seconds after its start from A , is (/).
- (ii) The probability that P is located at B 4 seconds after its start from A is (/).
- (iii) The probability that P is located at A 4 seconds after its start from A is (/).
- (iv) The probability that P is located at A 8 seconds after its start from A is (/).

II. Secondary Examinations to National Universities

[1] 1985, Tokyo Institute of Technology

There are three sets of cards, each set being composed of n cards numbered as 1, 2, ..., n . Now each of three persons holds one of these sets and randomly draw a card from his set and they compare the numbers on their cards. Then they gain scores by the number on his card according to the following rules. (Consider the two rules below).

Rule A. If there is only one person whose card has the strict maximum number, then he gains the score x which is equal to the number on his card and the other two persons gain nothing. If there are two or more persons whose cards have the maximum number, then any person gains nothing.

Rule B. In the Rule A, x is redefined as " x is the sum of the numbers on the cards of the other two persons".

Now, obtain the expectation of the score of each person according to Rule A and Rule B, respectively.

[2] 1985, Kyoto University

There are two coins. The probability that each coin falls with head up after being thrown is a and b , respectively. Let X be the number of the coins which fall with head up when we throw these two coins together. Thus the values of the random variable X are 0, 1, 2 and its distribution depends on a and b . Now consider the converse. Namely, when the distribution of X is prescribed as in the two cases below, answer whether there exist values of a and b which realize the given distribution and how many cases are possible when they do exist.

- (1) X follows the binomial distribution with

$$P(X = k) = {}_2C_k p^k (1-p)^{2-k}, \quad (k = 0, 1, 2), \text{ where } p (0 < p < 1) \text{ is a given constant.}$$

- (2) X follows the uniform distribution with

$$P(X = k) = \frac{1}{3} \quad (k = 0, 1, 2).$$

[3] 1985, Chiba University

The random variable X follows the binomial distribution $B(n, p)$, the variance being $8/9$. Suppose that $P(X = n - 1) = 8P(X = n)$. Then answer the following.

- (1) Obtain this $B(n, p)$.
- (2) Find $P(|X - m| > \sigma)$ where m is the mean value of X and σ the standard deviation.
- (3) Find the expectation of $X^2 - 3X + 2$.

[4] 1982, University of Tokyo

Firstly a dice is placed with its 1-spot face at the top. Then we change the position of the dice by repeated rotations. At each rotation, the angle of rotation is 90 degrees and the axis of rotation is a straight line connecting centers of a randomly chosen pair of square faces parallel to each other. Moreover, assume that any choice of possible axes is equally likely, and for each chosen axis any one of two possible directions of rotation is equally likely.

Let p_n, q_n and r_n denote the probabilities that the face of 1-spot is located on the top, on some of lateral sides and on the bottom just after the n -th rotation, respectively.

- (1) Find p_1, q_1, r_1 .
- (2) Express p_n, q_n, r_n in terms of $p_{n-1}, q_{n-1}, r_{n-1}$.
- (3) Find $p = \lim_{n \rightarrow \infty} p_n, q = \lim_{n \rightarrow \infty} q_n, r = \lim_{n \rightarrow \infty} r_n$.

[5] 1985, Gunma University

The point S moves in the region $\{(x, y) \mid 0 \leq x \leq 3, 0 \leq y \leq 3\}$. Starting from the origin, the movement of S is determined by the result of coin tossing. Namely, if the head occurs, then S moves by +1 in the direction of the x -axis, while, if the tail occurs, then S moves by +1 in the direction of the y -direction. If S cannot be translated in this manner within the region, then the coin is tossed repeatedly. This procedure is continued until S reaches (3, 3). Let the probability that the coin falls with head up and with the tail up be p and q , respectively ($p + q = 1$), and let X denote the number of tosses which was made until S reaches (3, 3).

- (1) Find $P(X = 6)$.
- (2) Obtain the distribution of the random variable X .

[6] Kyushu University

There are two jars A, B. The jar A contains a white balls and b red balls, while the jar B contains b white balls and a red balls ($a > 0, b > 0$). Firstly, we take out one ball randomly from the jar A, and return it there immediately after recognizing the color. If the ball taken turns out white, then we take out the next ball from the same jar, but if it turns out red, then we take out the next ball from the other jar. We repeat this procedure. Let p_n and q_n be the probability that the ball taken out at the n -th draw is white and red, respectively. Now answer the following:

- (1) Find the matrix $A = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$ such that

$$\begin{pmatrix} p_{n+1} \\ q_{n+1} \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} p_n \\ q_n \end{pmatrix} \quad (n = 1, 2, \dots).$$

- (2) Express p_n and then find $\lim_{n \rightarrow \infty} p_n$.

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COMMENTS ON PROF. FUJITA'S LECTURE

Tatsuo MIWA

Institute of Education, University of Tsukuba

It is a great honor and pleasure for me to be nominated as the first speaker to make a comment on Prof. Fujita's lecture. I am very grateful for Prof. Fujita and the organizing committee of this Conference, especially for Prof. Hida, Nagoya University.

As I will talk the present school system and the curriculum of school mathematics focussing on probability and statistics teaching of Japan this afternoon, I will skip this part and concentrate on Prof. Fujita's proposals.

First I would like to take note that I am a member of study group, whose director is Prof. Fujita, and agree with him in the fundamental philosophy of mathematics education, that is, the central goal of mathematics education is to cultivate mathematical intelligence, and in order to achieve the goal the followings are needed:

- a. The national curriculum should be of a flexible structure;
- b. Inspiring problems should be introduced in textbooks;
- c. Use of calculators should be encouraged;
- d. School use of personal computers should be developed.

Prof. Fujita's proposal on probability and statistics teaching is summarized as follows:

1. Statistics should be taught around at the 9th grade as a part of mathematics literacy;
2. The idea of testing hypotheses may be mentioned only as an example of application of probability distribution at the 11th or 12th grades;
3. Mathematics of senior secondary level should emphasize probability;
4. Use of calculators will reduce the difficulty in teaching normal distribution.

As to 1, I agree that statistics is one of very important mathematics literacy of the people in modern science-technology society and should be taught to all pupils irrespective of their future career. Therefore it is appropriate that statistics should be taught in the 9th grade – the final grade of compulsory education in Japan –.

The aim of statistics teaching is to develop pupil's ability to think statistically. By "to think statistically" I mean that pupils realize fully indeterminateness in an individual element and regularity in a population. In this point, present curriculum in junior high school mathematics is not satisfactory.

Not only meaning of frequency distributions and that of mean values but more suitable contents, e.g., correlation, time series, should be introduced in the grade.

As to 2, I agree in principle. It is because statistical inference in the present curriculum is questionable for me. Statistical inference, estimation and test, is taught in the subject "Probability-Statistics", though the Course of Study lists, as a content, an "idea of statistical inference" – sampling survey – but not statistical inference itself. Actually, teaching in class tends to stress the methods of how to estimate mean values and how to test hypotheses. This is too technical, and is not mathematical in its true sense, because basic properties of normal distribution upon which the methods are based are too difficult to deduce within mathematical techniques of senior high school and they depend on only experiments etc.

Therefore it is appropriate to mention test only as an application of probability distribution after full understanding of properties of them. However, to be noted is that an idea of sampling survey as a common-sense is necessary in everyday life, and we need to consider "when and how" to teach this idea.

As to 3, I should say yes and no. I say yes in the point that probability should be emphasized in senior high school mathematics. As statistical inference should be only an application of probability distribution, probability is naturally emphasized in senior high school mathematics.

But I should like to say no if to emphasize probability teaching leads to stress the determination of probability and as the results pupils would consider probability is only a ratio of possible outcomes. I would like to stress probability as mathematics, but not as only computation. Probability is a measure of certainty of indefinite phenomena. Stress should be made on this point – concept of probability –, and determination of probability be only an application of this concept.

Probability distribution should be emphasized more.

In addition, I would like to point out that situations dealt with in probability teaching should not be confined to mathematical models, such as coin tossing, die tossing, and lottery. Examples in various areas, e.g., physics, biology and economics, should be gathered and provided to pupils, so that they help pupils be confident that concepts of probability are pervaded and mathematics of probability has a strong power to solve difficulties in real world and it is not for a play or a game.

With regard to 4, it is certain that electronics facilities, such as calculators and computers are useful not only as teaching aids but also as learning aids. But how much it is effective, and how to utilize are, I think, now open problems. Prof. Fujita's proposal to use a calculator as a means of simplest numerical quadrature of $\exp(-ax)$ in the study of properties of normal distribution will be most valuable in probability teaching. I am sure an experimental teaching in mathematics class will validate his proposal.

**ON PARTICULAR ROLES PLAYED BY "PROBABILITY AND
STATISTICS" IN HIGH SCHOOL MATHEMATICS
— A COMMENT ON PROFESSOR FUJITA'S TALK —**

Takeyuki HIDA

Department of Mathematics, Nagoya University

In this short note I should like to emphasize particular, important roles and significance of the subject "Probability and Statistics" in connection with Professor Fujita's talk.

§1. Features

First let us observe that Probability and Statistics are occupying a special position among the high school mathematics and note that the subjects are most fitting for students to have intellectual challenge. The reasons may be illustrated by the following facts.

1. Many examples and problems discussed in the Probability and Statistics class come from the actual phenomena arising from daily life, so that students may feel familiar with them. Even, they will be keen to find applications of what they have learned.

2. To learn the subjects, not much prerequisite is necessary. Students who have missed opportunity to work mathematics hard, if any, will be given another opportunity to challenge mathematics. Namely, they can start afresh their study of mathematics by taking this opportunity.

3. Probability theory is quite logical. It is an established, indeed newly established, field in mathematics, however in the elementary level it is often thought of as a subject in which some ambiguity is involved. Such a misunderstanding might have come from examples of gambling, and of course this kind of misunderstanding of randomness should be expelled. Students must count or carefully compute the number of possible outcomes according to the given rule, or think of the probability under the given condition correctly, and so forth. Logical and systematic thinking is always requested to study probability theory.

4. Probability and Statistics have intimate connection with other fields of high school mathematics including combinatorial analysis, calculus and others. The subjects can therefore be taught by making reference to related topics in other fields.

§2. Proposal

As is emphasized in Professor Fujita's talk, Probability and Statistics have become important subjects in high school mathematics. In addition, systematic arrangement of these subjects has recently been made in the Japanese national curriculum of high school mathematics. Therefore it is now the time, for the teachers' side, to investigate how to teach the subject well.

The following facts are encouraged.

1. To find good exercises and problems. Tossing coin or throwing dice is certainly a good example, however it is referred to too often. Great variety of examples and exercises must stimulate the students' interest.

2. A bridge that connects Probability and Statistics should be built more closely, and the transition from Probability to Statistics has to be more natural.

3. Expectation or mean plays a very important role in the high school level. This concept should be understood correctly and be familiarized. For this purpose, again many good actual examples should be provided.

4. Gaussian distribution is really sitting in the center of the set of probability distributions. The importance of this distribution should be recognized in many ways; not only as a limit of normalized binomial distributions. It is an interesting idea, as Professor Fujita proposes, to use calculators to avoid the difficulty to understand the Gaussian density function $\frac{1}{\sqrt{2\pi}} e^{-x^2/2}$.

THE CONCEPT OF DEVIATION IN STATISTICS

Kazuo AKAO

Niwa High School

Approximately 90% of students at our high school enter universities or colleges every year.

Sections of "Probability" and "Statistics" were introduced only four years ago. "Probability" is taught 30 hours, while we teach "Statistics" about 10 hours. This is because of entrance examinations.

Recently in Japan, "deviation" is playing a very important part in consulting on choice of colleges for students.

We decided to let students understand deviation as a main purpose for "Statistics".

First, we taught the method of calculating standard deviation and deviation value. Then, students calculated them by electronical calculators. Each student calculated his own deviation, after he was informed of his own score of the examination.

After these studies, they wrote reports on them.

Essays on "Statistics"

We learned about average, variance, standard deviation, and deviation value in Statistics for several hours.

Today in Japan, "deviation abuse" causes various problems.

Write your opinion about it within 450 letters, at least 400 letters.

I think, at least, these contents should be taught to high school students both in a literary course and a science course.

The distribution table we used this time was almost normal in distribution as was expected. We are planning to choose non-normal distribution data next time in order to have good understanding of the deviation. Students will think of the notion more deeply.

Curriculum in Mathematics (Niwa High School)

Period in a week, a period: 50 minutes

		Science	Literary
The 1st Year	Math. I	6	6
The 2nd Year	Basic Analysis	4	3
	Algebra and Geometry	3	3
The 3rd Year	Differential and Integral	4	—
	Probability and Statistic	3	2
	Exercises	—	2

probability distribution

X	x_1	x_2	$\dots$	x_n	total
P	p_1	p_2	$\dots$	p_n	1

mean, expected, value : $m = E(X) = \sum_{k=1}^n x_k p_k$

variance : $V(X) = \sum_{k=1}^n (x_k - m)^2 p_k$

standard deviation : $\sigma(X) = \sqrt{V(X)} = \sqrt{\sum_{k=1}^n (x_k - m)^2 p_k} = \sqrt{E(X^2) - \{E(X)\}^2}$

deviation value : $50 + \frac{x - m}{\sigma(X)} \times 10$

TEACHING OF PROBABILITY AND STATISTICS AT THE SCHOOL LEVEL

Lennart RÅDE

Chalmers University of Technology
Gothenburg, Sweden

1. Why teach statistics at the school level?

The content of the mathematics curriculum at the school level has been very much discussed since the middle of the fifties. Numerous national and international conferences have been held to discuss this issue and several sets of recommendations have been formulated at such conferences about how to change the content of the mathematics curriculum. The following is a typical such recommendation which was formulated at the famous Royaumont Seminar in 1959, sponsored by OECD.

Elementary probability must be recognized as an appropriate part of mathematics in secondary schools.

- a) Statistical inference must be recognized as applied mathematics which contributes in an essential way to decision processes in the spirit of the "scientific method" basic to so many fields, both in the physical sciences and in fields of human behavior. Furthermore, it must be recognized that statistical reasoning is of growing importance in the field of public affairs.
- b) Suitable elementary instruction in probability and statistics must be introduced into the curricula of secondary schools.
- c) Suitable preparatory courses for teachers of these subjects must be introduced in normal schools and teacher-training institutions.

The recommendations above give some reasons why statistics should be introduced at the school level. In (Holmes, 1980) is found an extensive and penetrating discussion on why statistics should be taught at the school level. Holmes gives essentially six different reasons, namely the following:

- 1) Statistics is an integral part of our culture;
- 2) Statistics is an essential part of numeracy;
- 3) Statistics is an integral part of most peoples' work;
- 4) Statistics is an aid for personal development;
- 5) Statistics is an essential application of mathematics;
- 6) Statistics is an essential part of many subjects in the curriculum.

To these reasons can be added that statistics at the school level will give the foundation for further study of statistics after the school level.

2. The present situation

One would think that with the support of the recommendations above and numerous other similar recommendations and with the most relevant reasons above statistics should by now have a well established place in the curriculum. This is, however, not the case. The situation of statistics teaching in schools around the world has been surveyed by a committee established by the International Statistical Institute. The results have been presented in (Barnett, 1982). This survey consists of reports from different countries and the reporters very often express dissatisfaction with the present state of statistics teaching at the school level in their countries, see e.g. the reports in (Barnett, 1982) from Japan, Federal Republic of Germany, France, South Africa and Italy.

In other countries, however, the situation is somewhat better. For example in Hungary probability and statistics has been in and out of the curriculum since 1849, and at present these subjects are taught in all grades 1-8 of the primary school. In Sweden, probability and statistics is taught at least to some extent in all branches of the secondary school and some work is also done in the first nine years of the compulsory school. This is especially the case for the new curriculum for the economics, social science and humanities option, where at present probability and statistics is the largest specific content area of the curriculum.

In England most school curricula contain some probability and statistics. Of special interest is the material *Statistics in Your World* (1980), which was produced by the School Council Project on Statistical Education in Sheffield. Later an institute especially engaged in problems dealing with statistics teaching at the school level has been established in Sheffield. Its name is *Centre for Statistical Education* with Peter Holmes as director.

One can ask why the process of introducing probability and statistics at the school level has been so slow in many countries. This issue is discussed in (Råde, 1985b), where the following reasons for the slow process has been identified and discussed.

- 1) The foundations of statistics as a scientific discipline are under debate.
- 2) The uncertainty of where should be the suitable place for statistics in the school curriculum.
- 3) Very little is known about the didactics of statistics.
- 4) Lack of school teachers who are qualified to teach statistics.
- 5) Lack of appropriate teaching material for teaching statistics at the school level.

3. Some international activities

The International Statistical Institute (ISI) has through its Education Committee (chairman: Joe Gani) taken an active and leading rôle with regard to the improvement of statistics teaching at the school level. The Education Committee has established three task forces dealing with different aspects of the work of the Committee. Of special interest with regard to teaching at the school level are task force for statistics teaching at the school level (chairman: V. Barnett) and the task force for international conferences on teaching statistics (chairman: L. Råde). The first task force has published the report (Barnett, 1982), is publishing a newsletter and will publish an international directory of workers in statistical education. The second task force is responsible for the ICOTS conferences, the international conferences on statistical education, the first of which took place in Sheffield in 1982 and the second of which will take place in Victoria, British Columbia, Canada, in August 1986. This task force is also responsible for the

sequence of ISI round table conferences. The sixth such conference took place in Canberra, Australia, in 1984 with the theme *The impact of computers and calculators on teaching statistics*, see (Råde-Speed, 1985). The next round table conference will probably take place in Hungary in 1988 and the theme will be *training teachers to teach statistics*.

ISI also sponsors the journal *Teaching Statistics*. The aim of this journal is "to inform, entertain, encourage and enlighten all who use statistics in their teaching or who teach statistics". The journal appears three times a year (seven volumes have been published so far) and contains articles of great interest for all who teach statistics at the school level.

4. Trends in statistics education at school level

The following trends in statistics teaching at the school level have been identified in (Råde, 1985).

- 1) More emphasis on statistics than on probability with special emphasis on descriptive statistics including the use of exploratory data analysis.
- 2) Use of calculators and computers, both as practical and didactical tools.
- 3) Use of simulation.
- 4) Emphasis on applications and model building.
- 5) Use of project work.

Of these trends only nr 2 and nr 3 will be discussed here. With regard to trend nr 1 some illustrations of these trends are illustrated in appendix I by examples from some recent Swedish textbooks.

5. Use of computers

Calculators and computers are important in statistical practical work and should therefore be used also in teaching. But this is not the only reason for introducing these machines in teaching statistics at the school level. The computer can also be used as a didactical tool to make the students familiar in a concrete and challenging way with basic concepts and methods in probability and statistics. Computers and calculators can be used for demonstrating variability and stability in random experiments, for simulation, for statistical data analysis and for the simulation of statistical estimation procedures. Another interesting application is the calculation of basic probability distributions, where a programmable calculator with a few stored programs can be used instead of statistical tables. The following is a list of activities and problems which can be performed with calculators and computers. They are all described in

マイコンで学ぶ確率論

—— プログラム電卓用の確率の問題 ——

レンナット・ローデ

青木一芳 訳

and in (Råde 1976, 1981, 1983, 1985a)

- 1) Generation of random numbers between 0 and 1
- 2) Generation of random digits
- 3) Frequency test of random number generators
- 4) Poker test of random number generators
- 5) Simulation of random experiments like coins, spinners, towers and so on
- 6) Simulation of basic discrete probability distributions like the geometric distribution, the binomial and the Poisson distribution
- 7) Calculation with appropriate recursive algorithms of the binomial and the Poisson distributions
- 8) Simulation of Markov chains
- 9) Simulation of the exponential and the normal distributions
- 10) Simulation of random experiments in stochastic geometry
- 11) Simulation of the statistical estimation of an unknown probability
- 12) Simulation of the statistical estimation of the parameter A of the uniform distribution on the interval $(0, A)$.

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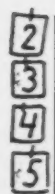
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Diagram 2

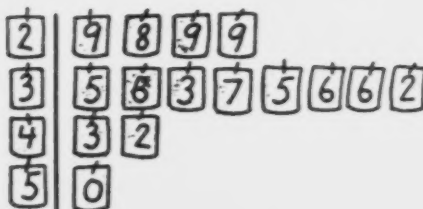
En dag skulle alla
hålla upp ett hårt papper
där de skrivit en förälders ålder.



Tiotal och ental klipptes isär.
En tiotal-siffra av
varje slag sattes upp.

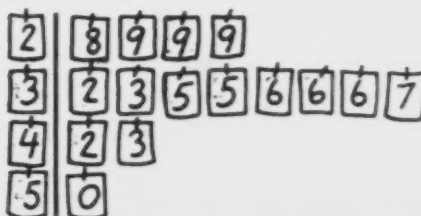


Sen fick en efter en
gå fram och sätta upp
sitt ental på rätt rad.



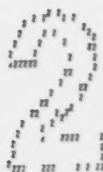
Åsa hade 43. Ser du var hon har satt sitt ental?

Det är bäst att börja
med det minsta entalet.
Eleverna fick ordna om.



Gör ett sådant här diagram i din klass.
Gör också ett över skolans personal.

From Johansson-Sandström-Dunkels, *Boken om enheter*, Gothenburg 1984. Shows the use of stem and leaf diagram in the elementary school.



Sanningen ligger alltid någonstans
mitt emellan.

Jacob Branting

Lådagram

2A Känner du din egen puls på nytt?

I avsnitt 1A analyserade vi pulsvärdena för eleverna i en klass med hjälp av stam-bladdiagram och fick resultatet som i fig 1.

Fig 1

Stam-bladdiagram över pulsen för en klass lärarkandidater.

total	ental
6 02	(2)
6 8	(1)
7 2224	(4)
7 6668	(4)
8 0000244	(8)
8 688	(3)
9	
9 68	(2)
10 0	(1)
	(25)

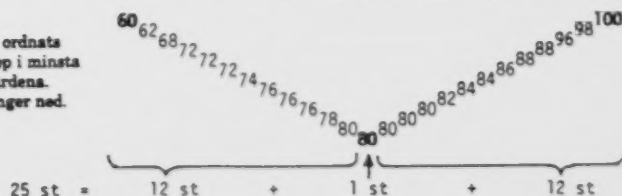
Vad har -stora massan- i klassen för pulsvärden? Hur ut-spridda är värdena?

Ibland innehåller ett stam-bladdiagram för mycket information för att ge överblick och möjlighet att direkt svara på såna frågor. Då behövs en enklare figur eller översikt som lyfter fram vissa egenskaper och utelämnar onödiga detaljer. En sån figur kan man få om man delar materialet i fjärdedelar. Vi ska göra denna delning i två steg. Vi delar först på mitten. Men allra först tar vi fram minsta värdet och största värdet, dvs -svans-värdena- i stam-bladdiagrammet, i vårt exempel 60 resp 100.

Nu tänker vi oss för ett ögonblick materialet -upphängt- som i fig 2. Vi ser att mittersta värdet helt enkelt är det 13:e värdet

Fig 2

Material som ordnats och hängts upp i minsta och största värdena. Medianen hänger ned.



29

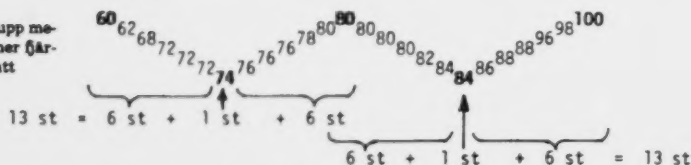
From Vännman-Dunkels, *Boken om kreativ statistik med EDA*, Gothia 1984. Shows EDA in teacher training.

från endera -svansen-. I vårt fall är det 80. Mittersta värdet kallas oftast med ett låneord för *medianvärdet*, eller kortare, *medianen*.

Ta nu medianvärdet och lyft upp det till samma nivå som minsta och största värdena som i fig 3. De två värden som hänger ned nu är nedre halvans mittersta värde och övre halvans mittersta värde. Man skulle kunna kalla dem fjärdedelsvärden, eller kortare, *fjärdedelar*. De märker ju ut nedre och övre fjär-

Fig 3

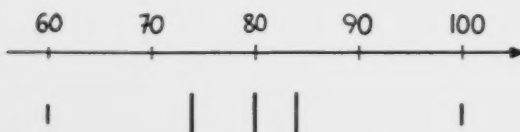
Om man lyfter upp medianen så kommer fjärdedelsvärdena att hänga ned.



dedelen av materialet. Lägg märke till att medianen kommer med i båda halvorna här. Det är praktiskt. Fig 3 ger en idé om hur man hittar fjärdedelarna. Det är bara att i stam-bladdiagrammet räkna in till det 7:e värdet, antingen från svansvärdet eller från medianen.

Fig 4

Fem viktiga värden uppritade på ett speciellt sätt intill en talaxel med skala.



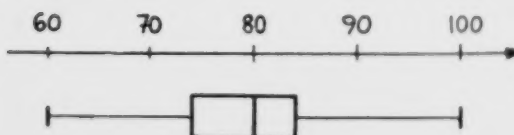
Vi ritar nu en talaxel försedd med lämplig skala. Under axeln markerar vi våra 5 framplokade värden:

- 60 = minsta värdet,
- 74 = nedre fjärdedelen,
- 80 = medianen,
- 84 = övre fjärdedelen,
- 100 = största värdet,

som i fig 4. Sen binder vi ihop markeringarna på det speciella sätt som visas i fig 5. Vi får då dels en -låda- som innehåller -mittersta hälften- av materialet, dels -morrhår- som förbin-

Fig 5

Lådagram. Strecket i lådan är medianvärdet.





From Göran Andersson, Lögn och sanning med statistik, Liber Läromedel 1982.

10.8 Vilseledande presentationer

Bilden är ett effektivt medel i reklam och opinionsbildning. Självklart används därför även diagram i försåtliga syften. Tidigare har visats hur man i kurvdiagram kan skapa helt olika intryck genom manipulering med skalorna. Det finns en annan sådan möjlighet, nämligen val av start- och slutpunkt. De här två diagrammen får illustrera.

Vem staplar rätt: SAF eller SCB?

Diagram 1. SAFs förklaring till Sveriges dåliga konkurrenskraft: arbetskraftskostnaderna är för höga (kurvan ligger över 100-linjen).

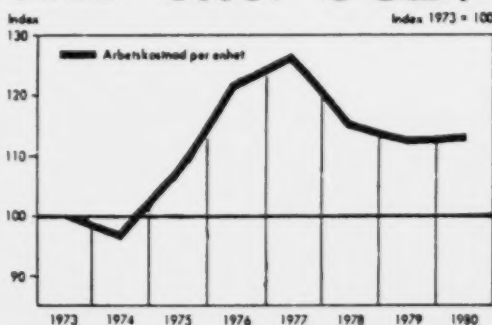
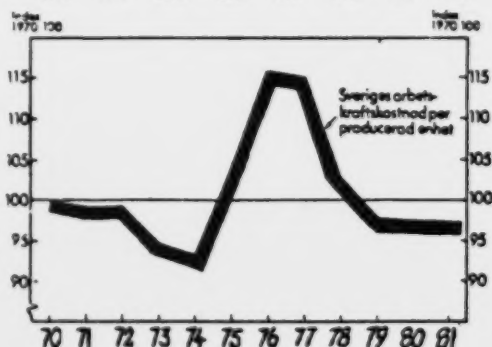


Diagram 2. En annan bild av samma sak, framtagen av SCB: arbetskraftskostnaderna i Sverige låg under merparten av 70-talet klart lägre än i jämförbara länder.



De här diagrammen diskuteras i en debattartikel i *Arbetet* 81-11-13. I bägge fallen har avsikten varit att illustrera hur arbetskraftskostnaderna i Sverige utvecklats i relation till jämförbara länder. Man tycks ha utgått från serier med något olika definitioner men skillnaden mellan diagrammen beror huvudsakligen på att det övre startar med 1973 $\hat{=}$ 100 och det undre med 1970 $\hat{=}$ 100.

FÄLLDIN

har krympt till hälften

Låget är nu rekordlångt för både Olof Wilander och Thorbjörn Fälldin. Vållarna överger nu i allt snabbare takt de två mittpartisterna.

Om trenden i den senaste ti-dans opinionsmätningar håller i sig går center och folkpartiet mot undergången som tidigare stora partier.

Center är i senaste SIFO-mätningen, gjord i början av juni, nere i 14,5 procent vilket är en förbättring av rekord-låget sedan 1973-74.

Center folkpartiet tappat mark. Par-tiet 7 procent i SIFO-mätningen är lägre än det rekordlåg, som nås av-tal till 1973-74.

Medan center går neråt har folkpartiet gått uppåt och har över viltare från de båda mitten-partierna.

Statsministerens har också börjat förlora viltare.

De tre senaste mätningarna av både Sifo och Sveriges opinionsundersökningar av viltare.

En riklig tillgång av viltare upp-lämnat om de tidigare partierne är att opinionsundersökningen på vil, som efter 1-2 dagar.

Om Sveriges viltare på de båda-partierna som var ut till tidigare till-lystningen.



1980:
14,5 procent



1973:
29 procent

Aftonbladet 80-06-26

Massmedia försöker ofta pigga upp de konventionella diagramtyperna med fantasifulla utsmyckningar. Det vanligaste är att staplar ersätts med avbildning av figurer som man associerar till i sammanhanget.

Dessa figurer brukar avbildas höjdproportionella. Centerpartiets 14,5 % mot tidigare 29 % blir en hälften så hög Fälldin. *Arean* blir dock enligt kända matematiska formler endast fjärdedelen och den här illustrationen ger — som väl avsikten också var — en överdriven bild av centerpartiets tillbakagång.

"sanningen" kan vara missuppfattning av frågan, glömska, okunnighet, prestige, förväntningsstress, egenintresse samt rädsla för att anonymitetsskyddet inte håller. Intervjuer har bl a den fördelen jämfört med postenkäter att risken för missuppfattning av frågan minskar.

Prestigebias. "Bias" betyder ungefär "ensidig riktning" och används i statistisk litteratur som alternativ benämning på systematiska fel. Vid intervjuer kan bias uppstå genom att respondenten av prestigeskäl inför intervjuaren avger andra svar än de sanna. Respondenten vill kanske framstå som mer allmänbildad, fördomsfri och välanpassad än vad han egentligen är.

Apropå mätfel, vad tror du om tillförlitligheten i sifferuppgifterna i följande utdrag ur Expressen?

EXPRESSEN ★ Torsdagen den 30 november 1978

Varannan elev i sjätte klass dricker alkohol — i nian 90 proc

Av ARNE WIRÉN

Varannan elev i sjätte klass dricker alkohol. Två av tio dricker starksprit. I nionde klass konsumerar nästan 90 procent av eleverna alkohol.

Det här är skrämmande siffror som tagits fram av Hem och Skola-föreningen i Bagarmossen.

Drygt 300 elever i årskurs 6-9 tillfrågades om sina alkoholvana.

— Visst dricker vi alkohol, svarade åtta av tio.

Var sjätte elev svarade att de regelbundet dricker starksprit. Minst 37 cl varje gång. 1-3 gånger per månad svarade 44 elever. 1-2 gånger per vecka svarade sex elever. Redan i årskurs sex dricker två av tio starksprit. I årskurs nio 90 procent.

Halv flaska

Men vanligast är öl och vin. Eleverna i nian dricker minst en halv flaska varje gång.

Enkäten gick ut i Bagarmossens skola och Brotorps skola.

Föräldrarna är oroliga: Stämmer det här?

— Ja, dessvärre är det nog riktiga siffror. Men det finns ingen anledning att tro att det är värre just i Bagarmossen. De här siffrorna skulle kunna komma från vilken skola som helst i Storstockholmsområdet, säger Herbert Davidsson på Stockholms skoldirektion.

Dennis Hansson, Hem och Skolas ordförande:

— Det är skrämmande siffror. Det är dags att vi som föräldrar tar vårt ansvar. Vi kan inte skjuta över allt på socialarbetare, politiker och polis.

BEST COPY AVAILABLE

ON RECENT CHANGES IN THE MATHEMATICS CURRICULUM IN NEW ZEALAND UPPER SECONDARY SCHOOLS

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1. Apologia

This paper reviews the background to some recent changes to the Mathematics prescriptions for the national examinations taken in the last two years of the New Zealand secondary schools programmes. Basic features of this programme, together with some local terminology, are set out in Figure 1. The changes which we shall describe have as one of their main components a significant increase in the work in probability and statistics included in the programme.

The Examination prescriptions are the responsibility of the New Zealand Universities' Entrance Board, which is charged, on behalf of all six New Zealand universities, with maintaining a common and appropriate standard of entrance to the universities, and with setting and conducting any examinations necessary for this purpose. The two examinations which will concern us are the so-called "University Entrance Examination", which is taken from Form 6 (year 11) and, the "University Bursaries Examination", taken from Form 7, (year 12). The former (if passed) entitles a student to enter a New Zealand University; the latter is used to determine the award of a basic tertiary assistance grant, and certain supplementary awards. The future of both these examinations is currently under discussion, but I shall not be concerned with the wider debate in the present paper.

In Mathematics the prescriptions are developed by a Steering Committee, which recommends draft proposals to the Board for circulation and comment from the schools and universities. Only after a general consensus has been reached are the new prescriptions approved, gazetted and (after 1 year's delay) adopted in the schools. The Steering Committee comprises three school members, three university members, a curriculum officer from the Department of Education, and the Secretary of the Universities Entrance Board. I recently had the privilege of completing a six-year period as Convenor of the Steering Committee, and as such presided over

Figure 1: New Zealand upper secondary school system

Years of Schooling	Median Age	Form Name	National Examination
10	15	Form 5	School Certificate
11	16	Form 6	University Entrance
12	17	Form 7	University Bursaries and Scholarships.

the developments which form the theme of the present paper. It is, I think, typical of the situation in New Zealand that of the membership of the Steering Committee, only the Department's curriculum officer had a professional involvement in curriculum development. The resulting prescriptions, which are set out for reference in the Appendices, no doubt reflect these limitations. My excuse for describing them here is the belief that the proposals, particularly where they concern the role of probability and statistics, raise issues of general importance.

2. The New Zealand Education System

To set the discussion in context, it may be desirable to make a few general comments about the educational system in New Zealand. A system of free and compulsory primary education has been in existence in New Zealand since 1877, and was gradually extended to cover the secondary curriculum during the first four decades of this century. Since 1944, there has been free, compulsory education for all children aged 7-15 inclusive. University education is open to all those who pass the University Entrance Examination, or who attain the age of 21. A system of grants is available to students who achieve a sufficiently high mark in the University Bursaries Examination, or whose actual performance at University reaches a sufficient standard.

The education system is broadly moulded along British lines, with perhaps a special input from the Scottish settlers in the south of New Zealand, who brought with them a traditional concern for education, with a distinct egalitarian bias. Their influence can be felt, for example, in the strong position of the state schools against the private schools – the latter catering for less than 13% of the students in the secondary schools, and a much smaller proportion of those in the primary schools. Education is funded on a national rather than on a regional or local basis. Uniform syllabuses are used throughout the country, and are developed primarily by the Department of Education with the assistance of syllabus committees including teachers and sometimes university or lay representatives. All six New Zealand universities are public (i.e. financed by the Government), but have considerable autonomy in regard to their teaching and research activities.

The fact that the qualification for University Entrance is generally obtained from Form 6 gives rise to a number of special features. The participation rate for Form 7 has generally been somewhat lower than the university matriculation rate even for those matriculating directly from the schools (see Figure 2); the 7th Form is seen as a desirable rather than a necessary stage for students wishing to proceed to University. Another feature is that far from all of those who are qualified to enter a university actually do so. In fact about half of those obtaining the University Entrance qualification matriculate directly from the sixth or seventh forms, representing about 12% of the total age cohort. Others matriculate later, and the percentage of the age cohort ultimately matriculating may reach 18% or even higher.

The mathematical standards reached by New Zealand children are disappointing by world standards, particularly in the early years of secondary school where there is a chronic shortage of qualified mathematics teachers. The relative standards seem to improve later in the secondary school, and the level of the Bursaries Examination in Form 7 somewhat between O-level and A-level in UK terms, though closer to O than to A.

As a general comment it seems fair to say that New Zealanders as a whole take some pride in the egalitarian aspects of their educational system, while remaining relatively indifferent to

Figure 2: Estimated participation rates – census years

		1966	1971	1976	1981
FORM 5	Male	67.7	76.8	83.5	84
	Female	67.6	77.8	87.6	87.6
	Total	67.6	77.3	85.5	85.9

FORM 6	Male	35.4	47	47.63	51.8
	Female	26.6	42.2	50.7	56.6
	Total	31.1	44.7	49.1	54.2

FORM 7	Male	11.1	15.6	15.9	17
	Female	5.2	9	11.3	15.6
	Total	8.2	12	13.6	16.3

1ST YEAR UNIVERSITY (Entering from School)	Male	14.1	17.8	13.9	12.7
	Female	8.0	12.3	11.4	11.5
	Total	11.1	15.1	12.7	12.1

About one-third of total university first enrolments are from students not proceeding to university directly from school. Rough estimates of the proportion of the age cohort who may ultimately matriculate are

15% 19% 18% 18%

Participation rates are based on figures interpolated from the census data and may contain errors of up to 1% or so.

the higher flights of intellectual or cultural achievement – both attitudes no doubt having their roots in the country's relatively recent pioneering history. The Maoris, who constitute some 15% of the country's population (though the figure is obscured by extensive intermarriage), have the same rights as other groups in the community, but, although represented in all walks of life in New Zealand, are not well-served by an educational system based almost exclusively on Western values and British traditions. This, and the future of the national examinations in the upper secondary school, are major issues of current public debate.

3. Changes in the Social Background

One of the most important factors prompting a re-examination of the mathematics curriculum in the upper secondary schools has been the massive increase in the numbers of children remaining at school into Forms 6 and 7. The phenomenon seems to be world-wide in character.

For Japan, for example, Fujita and Terada [1] state that "One of the most serious difficulties in Japanese high schools (10th-12th grades) is caused by a sociological factor, i.e. the very high ratio of advancements to high school education". Trying to outline the problems as we saw them in New Zealand in 1980, I wrote [2], "Many teachers at the 6th Form level face acute difficulties in teaching the present (mathematics) syllabus to classes which include many students who have only a poor understanding of the School Certificate programme and little interest in the topics emphasized in the U.E. course... The origins of these problems are to be found in the massive social changes which have affected the upper secondary schools in the last two decades. Starting in the early 1960's and continuing through to the present time there has been a steady increase in the proportion of the age cohort staying on into the 6th Form and now into the 7th Form as well. At the same time the number of girls taking the University Entrance Examination has caught up with and now slightly surpasses the number of boys".

An updated version of the figures referred to here are set out in Figure 2, which shows rough estimates of participation rates (% of the age cohort) for Forms 5, 6 and 7, and first year University courses.

By and large, these changes have taken place faster than the school programme has really been able to accommodate itself to them. Even now, the programme and style of teaching in Forms 6 and 7 continues to reflect the needs and concerns of students continuing to tertiary institutions, particularly the universities. School classes at these levels, however, are open to virtually the same ranges of abilities and interests as exist lower down in the school – the range may be even wider as the result of ten years' progressive fractionation during the preceding part of the school programme. As Fujita and Terada remark (*op. cit.*) the students in these classes "are no longer the elite children, and the high schoolteachers see the lowering or diversity of the aptitude of the students in their class".

The first effect of this situation on our deliberations within the Steering Committee was to produce a virtual impasse between the school representatives, who felt that the highest priority must be to develop a programme more in keeping with their perception of the interests and abilities of the students actually confronting them in the classroom, and the university representatives, who were concerned to maintain the entry standards in mathematics. A substantial part of the difficulty was perceived to lie in the dual role of the U.E. examination at Form 6, which was ostensibly geared to the function of setting an entry standard to the universities, but in fact dominated the teaching and curriculum for most of the 6th Form. Two factors helped to resolve this difficulty. Both sides were able to agree in putting forward urgent recommendations to change the location of the University Entrance Examination from Form 6 to Form 7 – a recommendation which had little effect at the time but may have helped to pave the way for the substantial changes in policy now under consideration. A second factor was a growing concern among the University representatives that the over-large and academic syllabuses might themselves be a factor contributing to a lowering of standards in mathematics on the part of students entering University. Ultimately both groups were able to agree on a substantially reduced syllabus (see Appendix I) which retained a brief introduction to calculus alongside a similar introduction to probability and statistics, and gave more stress to numerical work with calculators and to a consolidation of elementary algebraic and graphical skills.

In fact I strongly suspect that more than curriculum change is needed to address the problems introduced into the last two years of secondary school by the massive increases in the

participation rates. It may well require a reorganization of the whole school programme, as well as a substantial shift in attitudes towards the function of the last few years in secondary school. In the meantime, curriculum change offers a palliative measure which may help to remove the most acute sources of strain and to prepare the way for a more substantial reorganization.

4. Mathematics for Whom?

One of the most paradoxical features of the school situation is that, despite the mismatch between pupil and syllabuses referred to in the previous paragraph, which one might think would provide excellent grounds for mass defections from the mathematics courses at sixth and seventh Form levels, exactly the reverse has occurred. The proportion of students seeking to take mathematics in these forms has increased almost as fast as the total numbers themselves. Details for the New Zealand examinations are shown in Figure 3. The feature is particularly obvious in Form 6, where there is a single mathematics paper. The situation in Form 7 is more complex, and will be discussed further below, but overall a similar phenomenon has occurred.

Again the phenomenon seems to be world-wide in character. The reasons behind it remain to some extent a matter of speculation, though a number of circumstantial factors can be mentioned. Among them, the one that stands out is perhaps the rapid development of computers and their relatives, the hand-calculator and the microcomputer. All of these may be conceived of as broadly mathematical in character, and hence as enhancing the image of mathematics as a vital subject for modern young people. Such an impression would certainly be reinforced by the fact that many disciplines, trades, and professions have become increasingly quantitative in their requirements, even where they have been traditionally regarded as descriptive or non-quantitative. Courses in the social and biological sciences, business, industry, commerce, and not least in the computing and data-handling fields – all provide examples of such requirements. Fujita and Terada remark "The Japanese public is aware of the growing importance of mathematics and mathematics education. The parents encourage their children to study mathematics, sometimes, because they think that significant life and good job – opportunities in such a society require higher background, sometimes, because they simply believe that mathematics is a crucial subject at the entrance examinations to respected high schools and prestigious universities." Although the competitive entry to educational institutions is far less of a problem for New Zealand children than it is for their Japanese counterparts, similar remarks no doubt apply to many New Zealand children and their parents.

The motivations thus suggested are almost entirely pragmatic or even selfish in character. They have little to do with love of mathematics for its own sake. Indeed the interest in mathematics shown at school level is short-lived. While the numbers of students taking mathematics in Forms 6 and 7 has risen to unprecedented heights, the numbers continuing to the advanced study of Mathematics in the universities has actually declined. Some figures taken from enrolments in my own University are shown in Figure 4. The decline in 3rd year students is not marked, but the situation is far from reflecting the enhanced interest in mathematics at the school level.

Since the enrolment pattern in the physical sciences is similar, one is forced to the conclusion that most of the increase in the school population taking mathematics is due to students who are not proceeding along the traditional paths of the mathematics specialist, but are

moving into just those areas identified previously – the social and biological sciences, business and commerce, industry, computer or data-handling jobs, etc. It is therefore pertinent to enquire just what kind of mathematical skills are actually needed in such fields, and what kind of school programme might best prepare the children for their future work or study.

This is a question on which more factual information is needed. As a first step towards answering it, the Steering Committee conducted an informal survey of the mathematical techniques actually used in first year university courses across the whole range of university disciplines, whether quantitatively oriented or otherwise. It was somewhat reassuring to find that there were in fact a few disciplines – languages, history, most law subjects – which remained essentially free from mathematical contamination, at least at first year level. There were a surprising number, however, which made use at least of elementary mathematical and algebraic skills (substitution in and understanding of formulae; interpretation and construction of tables and graphs; elementary statistics, including calculation of means and display of data through histograms and other charts), and a substantial further group (the more quantitative courses in the biological and social sciences; courses in accounting and business administration; several in the earth sciences) which required additional skills such as some knowledge of linear equations and inequalities; some elementary coordinate geometry; an understanding of derivatives as slopes or rates of change; the elements of statistical sampling theory. Finally there were the traditional courses in mathematics, physics and engineering which required the full range of the school syllabus, including trigonometry, more advanced algebra and calculus, elementary vectors. The two main features which impressed us from this enquiry were the widespread demand for interpretive skills in dealing with graphs, tables, and elementary statistics; and the narrow range of courses requiring calculus-based techniques and trigonometry.

There are several morals which can be drawn from such considerations. Perhaps the first is that we should be cautious in our assessment of the current high demand for mathematics in the upper school. Many of the techniques referred to are ostensibly covered in the lower secondary school. For most students, more practice, particularly in the interpretive aspects, may be closer to what is needed than an extensive range of mathematical ideas. Part of the demand may also be due to mathematics serving as a surrogate for computing; in the future this part of the demand may be satisfied through the use of computers in the substantive disciplines, or by specific courses in Computer Studies. Another general point is that if the needs of all students are to be met, the mathematics programme must remain flexible, and responsive to considerations which have hitherto played little role in the determination of upper school mathematics curricula. Indeed, it is precisely against this background of changing social environment, and changing patterns in the uses of mathematics, that the case for an increased emphasis on probability and statistics stands out most clearly. It is not only that probability and statistics are directly useful in many of the user-subjects referred to above, but also that they provide excellent opportunities for building up the arithmetical, interpretive and graphical skills which seem to be so widely in demand. As one of my colleagues from Wellington Polytechnic put it to a group of teachers recently, " 'Numeracy through statistics' is not a bad policy".

5. Questions of Structure

Even if such points are broadly accepted, the problem remains of finding a manageable way of

incorporating them into the school programme. If there is a rising generation of secondary pupils for whom interpretive and statistical skills are of principal importance, there has been no real reduction in the numbers following the more traditional paths: both groups need to be catered for, alongside the parallel problem of handling the great diversity of ability and attainment already referred to. What structures will best meet these many-faceted requirements is very much a matter for debate and may well be strongly influenced by local circumstances. In this section I shall simply report some of the problems we encountered, and the solutions we put forward, without wishing to imply that the solutions proposed have any universal validity.

That some form of alternatives will have to be offered in years 11 and 12 seems almost inescapable, even if this only takes the form of streaming, or of grouping together students with similar interests. However, a formal core-plus-options scheme, particularly in the penultimate year, does give rise to some difficulties. One such proposal for the Form 6 examination was looked at in some depth, but finally was not adopted. The universities felt that the core material by itself (about 40% of total in this case) would be insufficient as a basis for students proceeding to university study, and that, with the remaining material as options only, they would have no firm basis for subsequent tuition. On the other hand, if specific options were nominated as being required for students proceeding to university, then many students would insist on taking them just for this reason and much of the value of the option scheme would be lost. Again to quote Fujita and Terada "According to our experience, any mathematics subject intended for low attainers can never become popular. Students and their parents reject explicit remedial courses. They prefer 'one and single track' ". A further difficulty with an options proposal in an examination context is that the examination itself becomes very complex, and raises difficult problems of maintaining equity between options. In the event, a "one and single track" proposal for Form 6 was finally adopted, and is attached in Appendix I.

The problems of structure were no less strenuously debated at the 7th Form level. They revolved initially around the future of the Applied Mathematics course, then generally taken by the better students as an adjunct to Pure Mathematics. It required a choice of two out of three options: Mechanics; Statistics and Probability; Computing and Numerical Mathematics. The Mechanics section had rapidly declined in popularity, while the other two made a combination which, particularly in the absence of a formal Computing Studies course, was increasingly in demand (see Figure 3). An interim proposal was put forward in 1981 to allow the Applied Mathematics paper to be taken more easily by itself, without Pure Mathematics. It was recognized, however, that a more substantial reappraisal of the whole structure at 6th and 7th Form level was necessary.

The nature of the structural changes finally adopted can be seen from Figure 4. They involve a single, integrated course at Form 6, including introductory sections in *both* calculus *and* probability and statistics, leading to two equal and parallel courses at Form 7, the first, "Mathematics with Statistics", aimed primarily at the "new users" of school mathematics discussed in the previous section, and the second, "Mathematics with Calculus" aimed at the more traditional hard sciences/engineering group. It is intended that each course should be capable of being taken either by itself or in conjunction with the other.

However, this was not the first structural proposal that the Steering Committee had put forward for the 7th Form. An earlier proposal involved a "General Mathematics" course, including statistics, to be taken by all mathematics students, together with an "Advanced

Figure 3: National entries in mathematics papers 1971-1984

University Entrance Examination								
Year	1971	1973	1975	1977	1979	1981	1983	1984
Total Entries	23,605	24,840	26,143	30,085	32,944	31,988	34,611	35,153
Maths Entries	12,950	14,971	19,688	23,773	26,893	26,489	29,564	29,387
Percentage of Total	55%	60%	75%	79%	82%	83%	85%	84%

University Bursaries Examination								
Year	1971	1973	1975	1977	1979	1981	1983	1984
Total Entries	4,696	4,979	5,010	5,823	6,325	6,814	7,203	7,893
Pure Maths Entries	4,329	4,637	4,849	5,335	5,862	5,985	6,147	6,177
Percentage of Total	92.2%	93.1%	96.8%	91.6%	92.7%	87.8%	85.3%	78.2%
Applied Maths Entries	1,490	1,296	1,691	2,371	3,244	4,022	4,791	4,948
Percentage of Total	31.7%	26%	33.8%	40.7%	51.3%	59%	66.5%	62.7%

Figure 4: Entries for university mathematics courses 1971-1984

Figures give full-year equivalents; students taking two courses or more are counted once for each course that they take. First year courses include all mathematics and statistics service courses, exclude computing science courses.

	1971	1973	1975	1978	1981	1983	1984
1st Year Mathematics Enrolments	1192	1098	1183	980	1033	1025	1115
2nd Year Mathematics Enrolments	200	242	163	202	187	225	196
3rd Year Mathematics	64	102	71	56	75	68	67

Mathematics" course to be taken in addition by abler students wishing to proceed with mathematics-based studies at university. This scheme was broadly acceptable to the Universities, but not to the teachers. They saw its effect as cramping the choices of the abler students, who would be forced to take two mathematics papers; as making unreasonable demands on the teacher requirements at small secondary schools; and, in some more general way, as offending the egalitarian spirit of New Zealand education. In short, they were against a hierarchical structure and preferred instead a system of two equal courses rationalizing and growing out of the existing "Pure" and "Applied" courses. Despite some initial scepticism as to whether such

a programme was feasible (because of problems of overlap and division of material), the Steering Committee was persuaded to rethink its approach and finally, to its own slight surprise, produced the proposals set out in Appendix II. These have now been generally accepted by both schools and universities. If anything, there are now more reservations on the university side, particularly on the part of more conservative applied mathematicians who see, not incorrectly, some running down of the traditional material in algebra, geometry and calculus on which their own application of mathematics is based.

6. Computing

A constant preoccupation throughout the discussions concerned the use of calculators, computers and microcomputers in the revised courses. In general terms, New Zealand has lagged behind a number of Western countries both in developing school courses in Computing Studies and in school uses of computing equipment. One reason may be that such equipment is largely developed outside New Zealand, is imported, and hence expensive.

Mathematics teachers had pioneered the introduction of computing in New Zealand schools, and were involved specifically with the popular Computing and Numerical Mathematics option within the Applied Mathematics course in Form 7. They were reluctant to drop this link with a topical subject which they saw as of considerable value and interest to their students, and which enhanced their own position within the schools. Nevertheless there was general agreement that topics such as programming, the principles of computer construction, and the impact of computers on industry and society generally, were not appropriately continued as part of a Mathematics course. In the meantime electronic calculators had become available to most students (there are still some problem areas – e.g. in the Pacific Islands) and were being cautiously advocated for use in examinations.

After considerable discussion, the Steering Committee finally became convinced that the proper attitude towards computing, at least insofar as it affected the mathematics programme, was to regard it as a tool which was extensively used in various parts of mathematics. It then followed that the school mathematics programme should include appropriate instruction in its use as a tool where relevant. This did not, however, imply, that the mathematics courses should concern themselves with the structure and principles of operation of computers, with commercial computing (file handling, etc) or even with programming, except insofar as students would need to be able to access a terminal or make use of simple packages or menu-driven programmes.

On the question of calculators, which are widely available, and useful in removing the need for arithmetical drudgery, it was felt that rather than leave to chance the extent to which students might be able to take advantage of such tools (and only a little acquaintance with the situation was needed to see that most students with scientific calculators were completely in the dark as to how to use many of the facilities on their machines) it would be better to provide specific instruction in their use and to include numerical questions requiring their use within the examinations. The Entrance Board has allowed calculators in their examinations for two years now, and no significant difficulties in regard to cheating or unfair advantages of some students over others have yet been reported.

Microcomputers are another matter, and practical considerations seem to rule out their use

in an examination context, at least in the near future. On the other hand it seems vitally important that students should have an opportunity at school to use such equipment for numerical and graphical work, for the manipulation, display, and basic analysis of data. Moreover, the software becoming available for such equipment mimics on a smaller scale the kind of software the students are likely to meet in a commercial or industrial environment; here also some practice in using packages, spreadsheets and similar software seems very desirable. Finally there is the important point that microcomputers provide the teachers with an instructional tool of considerable current and even greater potential value in the classroom.

It is this sort of philosophy which has been incorporated into all three prescriptions. Calculators are explicitly referred to in the Form 6 prescription and their use reinforced in the Form 7 prescriptions. The importance of practical work using microcomputers is stressed in both latter prescriptions. In the "Mathematics with Statistics" course there is also a 20% school-based practical component. The introduction of such a component is an important innovation in the mathematics programme at this level. It follows an earlier, unsuccessful attempt to introduce a 10% project component into the Form 6 course. This attempt was rejected by teachers on the grounds that it would impose a significant extra burden of work for only a small part of the assessment, and that finding and supervising projects for the large Form 6 classes would create major problems, at least without more experience and clearer guidelines. The later proposal moves slightly away from the project concept, allowing the teachers more flexibility in their approach to the practical work, and gives them the opportunity to develop experience in such work with the rather smaller, more manageable Form 7 classes.

For the time being, limitations on the extent to which microcomputers can be used are imposed by the difficulties of access to computing equipment by some students, for example those served by the correspondence schools. For this reason the requirements have been left at the stage where the practical work can, if necessary, be completed using only a programmable calculator. Undoubtedly the next few years will see further evolution of this aspect of the programme, as teachers become more familiar with the uses of computing equipment in the classroom, as the equipment itself evolves and becomes more widely available, and as computing continues to influence the uses and hence the content of mathematics itself.

7. Applications and Proofs

In this section, I wish to refer to what seem to me the two most difficult pedagogical questions arising in the mathematics programme in the final two years of the secondary school, namely, how to handle applications and proofs.

There is little doubt that simple, illustrative examples are a virtually indispensable accompaniment to the introduction of any new mathematical technique. The question that troubled us, however, concerned applications of a more extended character, the archetypal model being perhaps the extensive discussion of Newtonian mechanics in the traditional Applied Mathematics courses of British style. Courses on "modelling" represent a more recent and more flexible development along similar lines. Here again there is little doubting the value of such material when taught ably to a class of able students. The question facing the Steering Committee, however, was how and to what extent such material should be incorporated into general courses taught by average teachers to students of average ability. In this context the question is

not so easy to answer. The non-trivial discussion of an application is a higher-order skill: the student has to be familiar both with the context and with the mathematical technique required, and must be sufficiently confident with the latter to see how it can be used in a situation he or she has not necessarily confronted before. The ability to make such a sideways step, to match technique to problem, is not a common gift. For some students, the detailed study of a particular application can be decisive in illuminating a concept; for others it may have just the opposite effect – the context may be unfamiliar and confusing, the required simplifications may seem unnatural or unjustified, and the mental agility to make the link between application and technique may just not be present.

The form in which this question arises in the discussion of prescription content may be illustrated by many examples. Consider the discussion of, let us say, expected value. Some discussion of lotteries and insurance problems will not doubt illuminate this concept. What if anything, should be said about applications to insurance in the prescription? If it is excluded, the examiner will be confined to rather trivial questions on this particular application, otherwise he will face a barrage of complaints from teachers and students. If it is included, it runs the grave danger of becoming a topic learnt up in a stereotyped way for the sake of an extra mark or two. In any case it may or may not help to illuminate the concept of expected value – it could very easily serve to make it more obscure, particularly to students who had never had to consider questions of insurance before.

The new proposals are rather heavily committed to the principle that, as far as time permits, applications should be introduced and explored, alongside the mathematical concepts themselves. The prescriptions are supplemented by explanatory notes (not reproduced here) which indicate the depth to which particular applications might be taken, a number of applications being mentioned explicitly in the prescription. It must be admitted that the effectiveness of this approach, which represents a break with the tradition of handling the applications in a separate paper, remains to be tested, and is one aspect of the proposals that may need careful monitoring.

Proofs raise somewhat similar questions. It is a matter of concern to a number of university mathematicians that university students, even at second or third year level, frequently seem to have little idea of what constitutes a proof or how one should be set out. Premises are confused with conclusions; the logical progression of the argument is not clearly set out or even perceived; the discussion has no clear beginning and no clear end. Such a state of affairs can hardly be blamed entirely on the schools, but there is probably some truth in the argument that the demise of the traditional course in Euclidean Geometry, with its ponderous and sometimes even spuriously logical sequence of results, has left a vacuum in the school programme which has not yet been filled. It is not altogether surprising if large classes of average students, brought up on a diet of "New Mathematics", with its emphasis on concepts, understanding and exploration, have almost entirely lost the facility, which probably can be acquired only by extensive drill, to distinguish assumptions and definitions, to state them clearly at the beginning of a proof, and to carry through the proof to its conclusion. My own experience is that there is not only a lack of facility in but an actual hostility towards proofs on the part of many current students – proofs are seen as an unwarranted intrusion on the students' attempts to master techniques. Computers don't use proofs; computers do mathematics; why should students need proofs?

What can or should be done about proofs within the school programme? It does not seem to

me a question of topic. Probability and statistics, for example, offer just as good (if not better) opportunities for developing a logical argument as any other branch of mathematics. However, it would very likely be the death-knell of the topic as a school subject if teachers tried to develop probability in a purely deductive way from its axioms, at either sixth or seventh Form level. Indeed, the very case for its inclusion in the upper school programme is based partly on the fact that it is a useful, practical topic that represents a more appropriate field of study for the bulk of students than other more academic topics. Yet statistics, just as much as geometry or calculus, can be taught either in a way which emphasizes techniques and practical applications, or in a way which emphasizes the logical structure. The question really, as in the earlier discussion of applications, is just how far, in the school programme, should the logical aspect be taken?

It must be admitted that the proposed syllabuses are very timid in their approach to this question. It is accepted implicitly that proofs are desirable, and that mathematics owes its preeminent position precisely to the fact that its assertions are based on proofs, and are thus universal and divorced from any particular context or application. The prescriptions suggest that a cautious attempt should be made to introduce proofs whenever this seems natural and appropriate, and to emphasize the importance of their existence even where they are not explicitly developed. A few results, rather arbitrarily selected, are given specific mention in respect of proofs – for example it is suggested that the formula $\text{Var}(\bar{X}) = \sigma^2/n$ be derived from the rules for manipulating expectations, but that these rules themselves be left without the *requirement* of formal proof – the difficulty here being the extent to which calculus can be assumed within this particular course. (In fact this is a step backwards from the previous syllabus, where simple manipulations with density functions *were* required – but mastered only by a small proportion of the students!)

Even in retrospect, the best approach to these two questions – applications and proofs – remains nuclear to me. The effects of different approaches are not easy to measure quantitatively, and plausible arguments can be mounted in favour of quite opposing courses of action. It is in such respects, that teaching seems most an art, and that one is left most dependent on the skill and judgement of the teacher to find the right approach for the particular group of students in the class.

8. Implementation

There are, I think, two principal reasons why the incorporation of probability and statistics into the general school programme has not taken place more quickly. (The subjects have, after all been around for some time, and their inclusion in the upper school programme has been strongly argued for at least since the early sixties!) One reason is to be found in the composition of many mathematics curriculum committees, and (even more) of those groups responsible for setting the mathematics papers for entrance examinations to prestigious universities. Most often the membership of such groups is drawn from the mathematics, physical science and engineering disciplines traditionally served by the school mathematics programme; rather seldom is it thought necessary to consider the mathematical needs of the social or biological sciences, or even the commercial subjects. Indeed, when the purpose is to select the most able students for further study of mathematics, why should the mathematics departments not

emphasize those aspects which suit their own needs best? It is in such ways, however, that universities can exert a control over the school programme which, while perhaps unintended, may frustrate the attainment of more general educational goals. There are even some elements of a power struggle, between the supporters of the "old" and the "new" users of mathematics, and of course the "old" users have the historical advantage.

A second major reason for the relatively slow progress may be found in the attitudes and training of the school teachers themselves. Here, in my experience, it has not been a lack of recognition of the possible value of such changes that has hindered such progress but practical concern on the part of the teachers for their own lack of familiarity with the subject material and appropriate teaching methods. In New Zealand we have been fortunate in that the teaching of statistics in the upper secondary schools was pioneered more than a quarter of a century ago by enthusiasts (notably G. Jowett) who stressed from the outset the value of practical and experimental work, and emphasized the role of statistics in providing a direct link between the classroom and the world outside it. A detailed account of the development of statistics teaching in the N.Z. schools is given by Werry [3]. Even despite this long preparatory period, there are still many mathematics teachers in New Zealand who never studied statistics in their university courses, who have no or very little experience of teaching it in the classroom, and who feel very apprehensive as to their ability to cope with the new programmes at both sixth and seventh form levels.

To grapple with this problem, a major and coordinated effort has been required. A large number of short in-service courses have been mounted, first for the sixth Form and now for the seventh Form, which give teachers some impression of the style and content of the statistical material. They are also used to set out for teachers the resources available to them, and the kinds of teaching problems they will have to consider. At the same time the Department of Education has assembled groups of senior teachers, with experience in teaching the earlier statistics courses, to prepare detailed Guide Notes for all three new courses. These Guide Notes contain references to possible texts, suggested teaching sequences, explanations of particular concepts, and suggestions for practical work and the use of computers in the classroom. The Department is also preparing lists of software on teaching statistics, and other topics, and will have its own disc, in forms compatible with most of the more commonly used makes of micro-computers in the schools, available for teachers who may wish to make use of it. Finally, the New Zealand Mathematical Society has joined with local teachers' associations to prepare cheap, short-term texts which can be used during the first few years before appropriate commercial texts become available.

The fact that such steps were found to be essential even after the long preparatory period makes it clear that the practical problems of teacher-training cannot be taken lightly. It is not possible to take major steps towards reshaping the school programme unless this aspect can be tackled, and there exists already some teaching basis for the change. On the other hand no such basis is likely to exist until teachers are given some opportunity and incentive to gain experience and skill in teaching the new topics. In New Zealand the existing tradition and the relatively small, flexible and centrally administered education system have all helped to minimize the difficulty of the transition, while the Universities Entrance Board was able to provide a forum where agreed proposals could be worked out between school and university representatives. Even with these advantages the process has not been a rapid one.

9. Concluding Remarks

In this paper I have tried to present a review of the issues surrounding some recent revisions of the New Zealand mathematics curriculum in years 11 and 12, particularly those issues relating to the inclusion of a greater component of probability and statistics. It is my personal belief that these topics indeed warrant a larger place in the general school programme – a belief which I did not hold strongly to start with, but which has been largely developed as a direct result of my involvement in the revisions and the discussions relating to them. Despite the features peculiar to New Zealand, and the many shortcomings which the proposals doubtless contain, I hope they will have some interest for school programmes elsewhere.

The inclusion of statistics alongside of probability theory no doubt reflects a British rather than a continental view of the subject; but whatever one's view, the social changes which prompted these developments seem remarkably widespread, and lead directly to the concern that the mathematics programme must cater more explicitly for the very large numbers taking mathematics not as preparation for university studies in mathematics or the exact sciences, but as a support for study in the social and biological sciences, or for work in commerce, administration, business, and industry – the whole host of activities where numerical, statistical and interpretative mathematical skills are becoming increasingly important. In any such programme statistics seems bound to play an important role, and can hardly be studied in isolation from the work in probability, or without the backup of concurrent work in algebra and perhaps elementary calculus. Another change which reinforces this conclusion is the growing use of calculators and microcomputers in the school programme. Despite the major practical and tactical problems which must be overcome, I think changes of the kind outlined are worthy of very serious consideration, and shall be very glad if this paper helps to support the arguments of those seeking similar changes elsewhere.

In conclusion I should like to express my deep gratitude to the organizers of the Meeting for inviting me to participate and for giving me the opportunity of describing our experiences in New Zealand.

References

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APPENDIX I: UNIVERSITY ENTRANCE EXAMINATION

PREAMBLE

1. The prescription has the general aims of defining a level of mathematical understanding and a body of knowledge appropriate for pupils proceeding to tertiary level study, and within the constraint implied by this requirement, of providing a course that will also be appropriate to the needs of the wider group of pupils in the sixth form.
2. As corollaries to these general aims the prescription has the following more specific goals:
 - (a) To consolidate and extend the work of the School Certificate prescription in mathematics, especially the use of coordinate methods and statistics, and to provide a first introduction to the calculus.
 - (b) To provide opportunities for pupils to gain further experience in the use of graphical and computational techniques, including the use of calculators.
 - (c) To strengthen pupils' ability to manipulate algebraic expressions, to set out arguments in logical form, and to formulate problems in mathematical terms.
3. In teaching to the prescription, emphasis should be given to establishing a firm understanding of concepts and a master of basic techniques. Opportunities should be sought to foster pupils' awareness of the wide range of applications of mathematics as well as its intrinsic qualities and characteristics.

PRESCRIPTION

1. Algebra

Manipulation and simplification of algebraic expressions; factorisation; simplification of simple rational algebraic expressions.

Solution of polynomial equations in factored form; application of the remainder theorem to factorising polynomials.

Solution of quadratic equations by factors and the general formula; use of the formula with arbitrary numerical coefficients; nature of the roots of a quadratic equation; the discriminant.

Solution of pairs of simultaneous equations in two unknowns, with either both equations linear or one linear and one quadratic.

Problems leading to quadratic equations and to simultaneous equations of the above types.

Manipulation of indices, including fractional and negative indices; the concept of a logarithm; elementary properties of logarithms.

Sequences and series; the evaluation of the terms of a sequence from an expression for the general term; numerical summation of a finite series; formulae for the sum of a finite number of terms in an arithmetic or geometric series; the sum of an infinite geometric series.

2. Functions and Graphs

The form of the graph and simplest properties of the following particular functions:

$$y = ax^2 + bx + c; y = ax^3; y = a/(x + b); y = ax^{1/2}; y = |x|.$$

$$y = a^x, y = \log_a x.$$

$$y = a \sin(bx + c); y = a \cos(bx + c); y = \tan x.$$

Cubic polynomials in factored form.

Behaviour of the above functions under translations $y = f(x + a)$, $y = f(x) + b$, changes of scale $y = f(ax)$, $y = af(x)$, and reflections $y = f(-x)$, $y = -f(x)$. Accurate plotting of more general functions from a given formula. Recognition from the plotted function of qualitative features such as symmetry, periodic behaviour, maxima and minima, behaviour for large values of x and $-x$, discontinuities and infinities.

3. Coordinate Geometry

The equation and elementary coordinate geometry of the straight line; the distance between two points; the gradient of a line and mid-point of a line segment.

The intersection of two lines; condition for parallel and perpendicular lines.

The equation of a circle with centre at the origin. The intersection of a straight line with a circle, with the parabola $y = ax^2$, with the hyperbola $y = a/x$.

Simple applications of the above.

4. Trigonometry

Use of trigonometrical relations in solving triangles; sine and cosine formulae; determination of areas of triangles. Simple applications.

Circular measure. Arc length and area of a sector of a circle.

Extended definition of sine, cosine, tangent; graphs of the extended functions; solutions of $\sin(a + x) = b$, $\sin(ax) = b$, $\cos(a + x) = b$, $\cos(ax) = b$.

5. Calculus

The concept of a limit (informal treatment only). The derivatives as the slope of the tangent at a point on a curve.

The interpretation of derivative as rate of change. The derivative of a polynomial and of ax^n for real values of n . Applications of the derivative to the determination of intervals over which functions are increasing or decreasing; stationary points; problems on maxima and minima; problems on rate of change, including velocities and accelerations.

Antiderivatives.

The definite integral notation for the area under a curve; evaluation of definite integrals by antiderivatives; integrals of polynomial functions.

6. Probability and Statistics

Proportions, percentages, frequencies and relative frequencies.

Display of data, including histograms with equal intervals.

Estimation of probabilities from long-run proportions. Calculation of probabilities in cases of symmetry (equally likely outcomes); applications to simple coin and dice problems, properties of random numbers.

The normal distribution. Use of tables of the normal distribution. Reduction to standard form.

Calculation of sample mean, median, quartiles and standard deviation from ungrouped data. Sample values as estimates of the corresponding population values. Use of $s/\sqrt{n}$ as a measure of precision for the sample mean. The problem of bias. The use of random number tables to obtain an unbiased sample from a listed population.

Simple applications.

THE EXAMINATION

Candidates will be examined by a three-hour written examination. A limited choice of questions will be allowed. For the most part questions will test understanding of concepts and mastery of basic skills. Some small part of the examination may be used to test higher level abilities.

An approved published book of tables will be supplied to candidates. Calculators will be needed, and should have keys for the usual arithmetic operations; trigonometric, exponential and logarithmic functions; at least one memory which can be used for the accumulation of numbers and their squares.

APPENDIX II: UNIVERSITY BURSARIES EXAMINATION

Prescription for **MATHEMATICS WITH STATISTICS**. To be examined in 1986 and subsequent years.

PREAMBLE

1. The prescription has the general aim of defining a course of mathematical study appropriate for students interested in quantitative aspects of the biological and social sciences, medicine, commerce and administration, and in general any field where the collection, analysis and interpretation of quantitative data is important.

2. More specific goals of the prescription are:

- (a) to consolidate and extend sixth form work on basic algebraic, graphical and computational procedures;
- (b) to provide a first course in statistical procedures based on the normal distribution;
- (c) to extend students' abilities to tackle simple modelling and problem-solving situations, and to use calculators and computers to solve problems of a mathematical nature;
- (d) to develop students' awareness both of the wide range of applications of mathematics and of the reasons for its usefulness.

3. The use of calculators and computers is an integral part of this course. It is assumed that students entering this course will have had some previous experience with computers, micro-computers or hand calculators, to the extent that they can use a terminal or other equipment and call up a suitable programme or package. The knowledge of programming required will be at a very elementary level, and programming as such will not be examined.

4. The prescription should be viewed as a whole, with opportunities sought in all sections to reinforce the idea of modelling and to provide practice in problem-solving and applications. The division of the prescription material into sections has been adopted for clarity of presentation, and is not intended to indicate a teaching order or a rigid division for teaching and examination purposes.

PRESCRIPTION

1. Introduction to Computational Mathematics

Construction of algorithms for the fundamental computational processes, viz. repetition, summation, decision making (illustrated respectively by examples such as generating sequences by recurrence relations, summing series, evaluating functions defined piecewise and other problems of a similar level of difficulty).

Types of errors. Generation and propagation of errors. Examples of different types of errors occurring in practical computations.

2. Algebra

Solution of linear equations in two and three unknowns. Consistency and uniqueness of solutions. Problems leading to simultaneous equations.

The graphical solution of simple linear inequalities. Linear programming in two variables.

Permutations and combinations. Pascal's triangle. Elementary properties of the binomial coefficients. The binomial theorem for positive integral index.

Series; properties and formulae for the geometric and exponential series, including an informal treatment of convergence.

3. Functions, Graphs and Simple Modelling

The properties and graphs of the following special functions:

fractional and negative powers, x^a for rational and negative values of a ;

rational functions, $\frac{ax+b}{cx+d}$ (including the determination of asymptotes);

step functions and functions defined piecewise;

the exponential and logarithmic functions, e^x and $\ln x (= \log_e x)$.

The derivative and integral of polynomial functions, the function x^a (for rational a) and the exponential function e^{ax} for real values of a . The derivative of the logarithmic function.

The exponential function as the solution of the differential equation $y' = ay$. Applications of calculus to geometric growth and decay curves (including population growth and decay, interest and inflation problems) and to simple problems of optimization.

The use of logarithms for linearizing power law and exponential curves.

Numerical solution of non-linear equations $f(x) = 0$. Bisection, secant and Newton-Raphson methods.

4. Probability and Statistics

Basic concepts: sample space, event, random variable.

Disjoint events. Combining probabilities; conditional probability; independent events. Use of branching diagrams. Applications to simple games of chance, sampling with and without replacement.

Discrete random variables. The expected value of a discrete random variable as the idealization of a long-run average. The binomial and Poisson distributions and their applications. The normal approximation to the binomial.

Display of univariate and bivariate data: in particular "stem and leaf" plots, bar graphs, histograms, scatter diagrams, "box and whisker" plots.

Sample mean and standard deviation. The use of Σ -notation in formulae such as

$$\sum_{i=1}^n (x_i - \bar{x})^2 = \sum_{i=1}^n x_i^2 - n\bar{x}^2$$

The use of median and quartiles as alternatives to mean and standard deviation.

Revision of the normal distribution, reduction to standard form, use of tables. The idea of the central limit theorem. Formulae for the expected value and variance of a sum or mean of random variables.

Sample estimates as point estimates of a population parameter. The standard error of an estimate; confidence intervals based on "the 2-standard error rule". Standard error and (large-sample) confidence intervals for the population mean, the population proportion, and the difference between the means of two populations. The ideas of bias and random sampling error in practical sampling situations.

Introduction to the idea of a statistical test; null and alternative hypotheses; test statistic; significance level. Large sample tests for the mean, proportion, and difference of two means.

5. Internally Assessed Practical Component

A set of one or more pieces of practical work, presented in written form, and internally assessed out of a maximum of 20%.

This section should provide an opportunity for candidates to demonstrate their ability in respect of selected practical aspects of the syllabus which are not appropriately assessed within the limitations of a 3-hour formal examination. These aspects include for example the ability to use computers effectively; to design, implement and analyse statistical projects or simulation experiments; to develop special purpose programmes and other software; etc. They include also the ability to present such work in a clear, informative and attractive manner.

ASSESSMENT

Candidates will be assessed on the basis of a 3-hour written examination, worth 80% of the marks, and the internally assessed practical component, worth 20% of the marks. The final mark out of 100 will be obtained by using the written examination as a group reference standard for moderating the internally assessed marks for the candidates from each school.

The internally assessed mark, together with a brief title or description of the work undertaken, must be forwarded by school principals on the University Entrance Board's approved school candidate list, so as to be with the Secretary of the Board, PO Box 12-348, WELLINGTON NORTH, by the Monday following the tenth day of November.

THE EXAMINATION

The examination will be based on Sections 1-4 of the prescription, these sections carrying in order the approximate weightings 15:15:20:50. A limited choice of questions will be allowed. Questions may be set to combine material from different sections of the prescription. For the most part questions will test understanding of concepts and mastery of basic skills, but some part of the examination may be used to test higher order skills such as problem solving ability and the ability to develop simple proofs or riders.

Standard tables and formulae will be supplied. Candidates are required to supply their own calculators, which they will be permitted to take into the examination, and expected to use as appropriate in questions of a numerical character. Calculators should have keys for the usual arithmetic operations, for the exponential, logarithmic and square root functions, and for accumulating a series of numbers.

THE INTERNALLY ASSESSED PRACTICAL COMPONENT

Assessment should take cognisance of the level of initiative and originality shown by the candidate; the candidate's ability to design, carry through and describe an appropriate study; his or her practical competence in handling computers or similar computational aides; the quality of presentation of the material.

All work undertaken as part of the internally assessed component should be clearly written up and entitled, and should be available for moderation and inspection purposes at the end of the year.

UNIVERSITY BURSARIES EXAMINATION

Prescription for **MATHEMATICS WITH CALCULUS**. To be examined in 1986 and subsequent years.

PREAMBLE

1. The general aim of the prescription is to define a course of mathematical study appropriate for students with a major interest in the continued study of mathematics, the physical sciences, engineering, and in general any field where analysis is an important tool.
2. More specific goals of the prescription are:
 - (a) to continue the development of the main stream of mathematics, with particular emphasis on calculus;
 - (b) to consolidate and extend students' facility with basic algebraic, graphical and computation skills;
 - (c) to extend students' ability to formulate problems in mathematical terms and to use their mathematical knowledge in applications;
 - (d) to advance students' understanding of the nature of mathematical reasoning and the correct use of symbols, and their ability to construct and set out methodically the steps in a logical argument recognising the need for clearly stated assumptions and definitions.
3. The prescription should be viewed as a whole, with opportunities sought in all sections to reinforce basic techniques, to provide practice in problem-solving, applications, and the use of calculators. The division of the material into sections has been adopted for clarity of presentation, and is not intended to indicate a teaching order or rigid divisions for teaching or examination purposes.

PRESCRIPTION

1. Algebra

Quadratic equations. The method of completing the square for quadratic functions. Revision of the factorization and remainder theorems (with proofs). Equations with real and complex roots, in particular complex conjugate roots.

Real, rational and complex numbers. Manipulation of complex numbers in $a + ib$ form. Representation of complex numbers by Argand diagrams. The modulus and argument of a complex number. Representation and manipulation of complex numbers in polar form. De Moivre's Theorem. Solution of equations $z^n = a$ for positive integral values of n , and a in the form $r \cos \theta$.

The limit of a sequence of real numbers. Convergence of a series in terms of the convergence of its partial sums. The arithmetic, geometric, exponential and logarithmic series.

The binomial theorem for positive integral index.

2. Co-ordinate Geometry

Equations of a line and circle (including the general equation of a circle). Equations of ellipse,

parabola and hyperbola in standard form.

Examples of curves given in parametric form, with applications.

Conditions for tangency to a curve. Finding the equation of the tangent and normal to a given curve, at a given point on it, using the derivative.

3. Trigonometry

The inverse trigonometric functions $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$, and their graphs. General solutions to the equations $A \sin(x + B) = C$, $A \cos(x + B) = C$, $A \tan(x + B) = C$.

The reciprocal trigonometric functions $\sec$, $\cot$, cosec and their graphs. Relationships between these functions.

Compound angle, double angle and addition formulae, with proofs. Simple trigonometrical identities.

4. Calculus

Formal definitions of functions and mappings. The range and domain of a function. Inverse and composite functions. The exponential and logarithmic functions.

The idea of continuity and limit value of a function. The derivative defined as

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Differentiation of sums, products and quotients. The chain rule for differentiation of composite functions. The derivatives of polynomials, rational functions, exponential and logarithmic functions, the trigonometric functions. Derivatives of functions defined parametrically.

Derivatives of second and higher order. Applications to the properties of curves (maxima and minima, points of inflection, regions of increase and decrease). Problems on maxima and minima. Problems involving variable rates of change, including simple applications in kinematics.

Integration. Antiderivatives and definite integrals. Integration by substitution in simple cases. Integration of polynomial, exponential and trigonometric functions, and of rational functions, $\frac{ax+b}{cx+d}$. Applications to areas and volumes of revolution. Introduction to numerical integration (trapezoidal rule and Simpson's rule).

The idea of a differential equation. The solution of differential equations of the form $y' = f(x)$, $y'' = f(x)$, and of equations with variables separable, in general form or with initial conditions. The equations of exponential growth and decay, with applications such as Newton's law of cooling, chemical rate reactions, etc. Simple applications to kinematics. Introduction to numerical solution of differential equations.

ASSESSMENT

Candidates will be assessed by a 3-hour written examination. A limited choice of questions will be allowed. Questions will test understanding of concepts, mastery of basic skills, the ability to set out logical arguments and to solve straightforward problems. Numerical questions involving the use of calculators may be set on any part of the syllabus.

Questions involving extensive algebraic or numerical manipulations, or a considerable background in physics or other area of application, will be avoided.

The sections will carry roughly the weights 25:15:15:45 in the examination.

Standard tables and formulae will be supplied. Candidates are required to supply their own calculators, which they are permitted to bring to the examination. Calculators should have keys for the usual arithmetic operations, for the exponential, logarithmic and trigonometric functions, and for accumulating a series of numbers.

REFORMS IN MATHEMATICS EDUCATION FOR SECONDARY SCHOOLS IN KOREA — WITH REFERENCE TO STATISTICS AND PROBABILITY —

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Introduction

In this paper an attempt is made to see how mathematics education for high schools have developed in Korea through a series of reforms conducted since 1945, and what its main features are with special reference to statistics and probability.

Korean mathematics education has undergone four reforms since liberation in August 1945. We first established our system in March 1946.

First Reform	April 1954
Second Reform	February 1963
Third Reform	August 1971
Fourth Reform	January 1981

The current system of high school mathematics (10th to 12th grades) consists of five branches:

- (A) Set theory
- (B) Algebra
- (C) Geometry
- (D) Analysis
- (E) Statistics and Probability.

Despite these reforms in mathematics education, statistics had not gone through any meaningful changes until the last reform in January 1981.

The following chart shows the contents of the old and current curricula.

Modernization and Reaction

In keeping pace with the worldwide tendency toward the modernization of mathematics education, the third reform was made in 1971. The curriculum worked out was a knowledge-centered one in which structure and logic were stressed.

In the latter half of the 1970s, however, the curriculum began to draw the fire of critics. So many countries of the world began to move away from the structuralistic knowledge-centered mathematics education and proceed gradually to humanism-centered one. In the latter an emphasis was placed on the development of the ability of the students to cope with the problems arising out of social phenomena around them.

In Korea, following this widespread trend, the fourth reform was made in 1981, and the

Chart

Old Curriculum	Current Curriculum	Remarks
(i) permutation, combination	(i) permutation, combination	1. In the old curriculum, part of statistics (estimation and test) were missing, while the current one treats them rather extensively.
(ii) probability (concept of probability the calculation of probability, expected value)	(ii) probability (concept of probability the calculation of probability, expected value)	
(iii) statistics (variance, standard deviation)	(iii) statistics (mean and standard deviation, probability distribution)	2. In the new curriculum the calculation of probability is taught within the framework of finite set.
(iv) binomial distribution normal distribution	(iv) binomial distribution standard distribution	3. The set theoretic method is applied with symbolic method within the framework of finite set in the current curriculum.
(v) sample survey	(v) sample survey	
	(vi) stochastic (estimation and test)	

In the reforms, the junior high school curriculum (for 7th to 12th grades) was considered prerequisite to that of senior high schools (10th to 12th grades).

Contents of Junior High School Curriculum

7th grade	The concept of probability expected value
8th grade	A. Collection of data frequency distribution relative distribution cumulative frequency histogram B. Representative (mean) temporary average degree of scattering (variance, standard deviation)
9th grade	A. correlation diagram correlation table B. Sample survey sample, population, sampling

schools have been using the new textbooks compiled under the reform since 1984.

In the fourth reform, the defects inherent in the old curriculum were either removed and/or corrected. Among the defects were an structuralist approach, the shortage of teaching hours due to an excessive emphasis on guidance, lack of understanding of the textbooks on the part of the teachers, teaching of highly conceptualized set theory, which were all apart from actual life.

As the result of the reform, the units of teaching were reduced in number, an optimum level of teaching set and life-centered mathematics infused into theory-heavy mathematics to a considerable extent. Consequently, statistics and probability was emphasized more than ever.

This will be summarized as follows:

1. The contents and concepts of the units are selected to fit the stages of growth of the

students and to make them interrelated between grades. Thus, the acquisition of basic knowledge is emphasized.

2. Mathematical structure and logic are not over-stressed.
3. An ability to solve the problems related to social phenomena empirically is highly regarded.
4. A step-by-step approach is taken in the compilation of the new textbooks to afford the students to learn basic concepts thoroughly.

The following guidelines were adopted to lessen the burden of the students while keeping alive the basic principles of the modernization of mathematics education and to enable the students in three major categories – the humanities, natural sciences, and social science – to learn basic units as much as possible:

1. The contents of textbooks and the methods of presentation should be so selected as to reflect the reality of life while keeping pace with the worldwide trends in mathematics education.
2. In the selection of the contents and method of presentation the process of mathematical development should be duly taken into consideration, but the main stream of the curriculum should be made systematic taking into account the structure of this discipline.
3. The textbooks should be divided into *Mathematics I* and *Mathematics II-1* (for those students whose undergraduate majors will be in the humanities and social sciences), *Mathematics II-2* (natural sciences) so that the students could better prepare themselves for the major subjects to be pursued in higher learning institutions.
4. Drawing on the understanding of the basic concepts, principles, and laws in mathematics and their interrelationship, the students should be taught to think logically about the daily phenomena in their life environment and to cope with them.
5. Knowledge and functions of mathematics should be systematized, and the students should be taught to utilize them in the logical solution of the problems they encounter in their daily life.
6. Those units related to other courses of study should be taught earlier unless it is difficult to do so.

The units in the high school textbooks that deal with statistics are probability and statistics included in common in *Mathematics II-1* and *Mathematics II-2*.

The objectives of teaching, the points to be stressed in contents, and evaluation are presented in these textbooks.

The students should acquire the knowledge of basic concepts and laws of probability and statistics, together with the ability to deal statistically with the data presented.

This will enable the students to understand how abstract concepts of mathematics are applied to the solution of the concrete problems they face in their daily life.

Thus, they will come to know how widely mathematics is utilized in problem solving not only in sciences and engineering but also in the humanities and social sciences and in predicting the future. In short, they will be convinced of the need and usefulness of mathematics in our life.

The teachers are cautioned to be mindful of the following in the instruction of statistics.

The meanings of estimation and test should be fully grasped and be put to practical use.

Since the emergence of stochastic statistics in the twentieth century, estimation and test

have been basic to the application of statistics to many fields. Statistics as a field of academic problems in our daily life.

The points to be reckoned with care in the instruction of test are as follows:

1. In the sequence of teaching, pre-unit evaluation should be made diagnostically to confirm the achievement in the preceding unit and to proceed to the next unit.
2. In teaching concepts, principles and laws formative evaluation should be conducted to confirm whether the students have fully understood them, and whether supplementary teaching is required.
3. The teachers are instructed to work out the evaluation items based on the clearly defined objectives to help the students develop problem-solving ability.

Evaluation — diagnostic, formative, and comprehensive — is designed to confirm if the students have completely understood what was taught, to adjust teaching schedule, and use the findings for other purposes, the ultimate goal being the promotion of the effect of teaching.

Statistics being considered important, it is treated as an independent course of study in high schools. Because of abstract concepts used in the textbooks, numbering five but almost identical in contents, however, there still exists a gap to be filled between theory and practice-application in daily life.

These five textbooks follow the guidelines set down by the Ministry of Education and were subject to its approval for publication.

Problems

The biggest problem in Korean mathematics education, particularly in probability and statistics has arisen from the following:

1. Even though our educational system and social conditions are different from those of the U.S. and Japan, Korean education has been greatly influenced by these two countries, either through foreign aid from the former or because of traditionally close relations Korea maintains with the latter.
2. We have a national qualification examinations for entrance to higher learning institutions, and the method and the type of problems of the examination virtually decide the substance of mathematics education in high schools. Unfortunately, there exists a gap between ideals of education and the demands of students.

As multiple choice system is used in the examination, the instruction in probability and statistics has been hampered greatly, becoming rather formalistic and mechanical.

When we compare the curriculum of high school mathematics teaching related to probability and statistics with that in the United States and Japan, we notice a considerable difference in levels: the Korean level is much higher. This does not imply, however, that its quality is also high. Korean mathematics education tends to be more formalistic than substantial because of the artificially set high level of instruction and the multiple choice style qualification examination.

Conclusion

In order to eliminate the defects in the teaching of probability and statistics in Korea the following measures are considered useful:

1. Reform of the college entrance examination system;
2. Lowering of the levels of curricula;
3. Emphasis of closer link with daily phenomena rather than academic teaching.

ON A METHOD OF CHOOSING ONE PERSON OUT OF MANY

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Let us suppose n persons be going to choose a person out of them. In Japan, people usually put out fingers in such cases and so do they in Russia. The Russian method of choosing a person by putting out fingers is as follows: Firstly, they stand around a circle and each of them puts out x_i ($i \in \{0, 1, 2, 3, 4, 5\}$, in case of using one hand or $\{0, 1, 2, \dots, 10\}$ if the both hands are used) fingers at random all at once. Secondly, they find out the total number of the fingers $S = x_0 + x_1 + \dots + x_{n-1}$. Lastly, starting with 0 to S , they count from the previously appointed person P_0 clockwise or counterclockwise, and the person at whom the count is finished is the chosen person.

In November 1981, at the *young* mathematicians' festival of the Soviet Union, a high school student Mitya Nogin presented a report discussing whether this choice is fair or not. This report is based on his considerations.

Suppose each person can put out at most m fingers. As an example, we consider the case of two persons and using both hands, that is $n = 2$ and $m = 10$. In this case, the person P_0 is chosen if and only if the sum of the put out fingers $S = x_0 + x_1$ is an even number. The total number of the cases of the even S 's is equal to the sum $\sum_{k=0}^{10} (2k+1)$ (the number of the representations of $2k$ as the sum of two nonnegative integers) $= 1 + 3 + 5 + 7 + 9 + 11 + 9 + 7 + 5 + 3 + 1 = 61$. Similarly, the number of the cases of the odd S 's in which the person P_1 is chosen turns out to be $2 + 4 + 6 + 8 + 10 + 10 + 8 + 6 + 4 + 2 = 60$. Of course, we may regard these $60 + 61 = 121$ cases to be equally probable. Therefore, the choice is not fair and the person P_0 has a little advantage over the person P_1 .

Let $\Pi_m^n(N)$ be the number of the representations of N as the sum of n nonnegative integers up to m in which the order of the summands is relevant. Further, let Π_i be the number of the cases in which the i th person is chosen and $\Pi = \Pi_0 + \Pi_1 + \dots + \Pi_{n-1}$. Obviously, $\Pi = (m+1)^n$ and it is necessary for the choice to be fair that Π is a multiple of n .

Remark. If $n = m + 1$, then we can show that the choice is fair i.e. every participant has the same chance of choice under this method.

Mitya Nogin, making use of the facts that $\Pi_m^n(0) < \Pi_m^n(1) < \dots < \Pi_m^n(\lfloor mn/2 \rfloor)$ and $\Pi_m^n(N) = \Pi_m^n(mn - N)$ for $0 \leq N \leq mn$, stated that if m is *odd* (putting out of fingers by one hand) the persons with the largest chance of choice are the ones opposite to P_0 and if m is *even* (putting out of fingers by both hands) P_0 himself has the largest chance of choice and the bigger n or the smaller m implies the bigger difference of chances among the participants.

When $m = 5$, the computer calculation shows that the ratio of the maximal value of Π_i to

the minimal value of Π_l increases with n and the ratio exceeds 2 when $n = 33$ and 3 when $n = 43$.

Reference

D. Nogin, Who will fetch the lemonade?, *Kvant*, Oct. 1982, 19-21. (in Russian)

ENJOYABLE LESSONS OF PROBABILITY THEORY

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Hibiya High School, Tokyo

We think that probability theory is one of the most important subjects of high school mathematics.

It is a useful key to our scientific understanding of the world. It helps to teach us that there are hidden laws behind various phenomena.

Well, how should we teach our students the probability theory?

We don't want to teach them one-sidedly from a teacher to his students. But we hope to offer our students materials which will arise their interests and to give enjoyable lessons in which they discuss, make experiments, repeat trial and error and finally rediscover the theory of probability by themselves.

If students understand the basic concepts of the probability theory and can apply them to various problems in their daily life, our purpose could be said to be successful.

Based upon this idea, we – 10 members of the Association of Mathematics Instruction (AMI) – prepared plans for enjoyable lessons of probability.

The following are several of the plans. I am glad to show you our plans and to hear your opinions about them.

1. How should students understand the concept of probability?

The concept of "probability" is very important. How should students understand this concept? We think that it is necessary to understand this concept as the quantity which really exists in the world. Our methods are as follows.

We let students try to throw a "Saidota" (a non-cubic die of 3 cm $\times$ 3 cm $\times$ 3.5 cm) 30 times and count the frequencies of "Ace". For example they will get the following data.

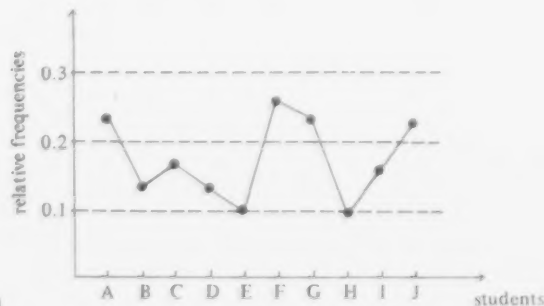
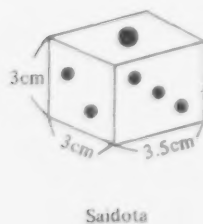


Figure 1

Now we ask students, "From these data how do you guess the divergence will become, when you increase the numbers of trials?"

Some students will guess that the divergence will converge, while others will not.

Experiments of 120 trials and 480 trials will be as follows. Through these experiments students will understand that probability is not an imaginary quantity but a real one.

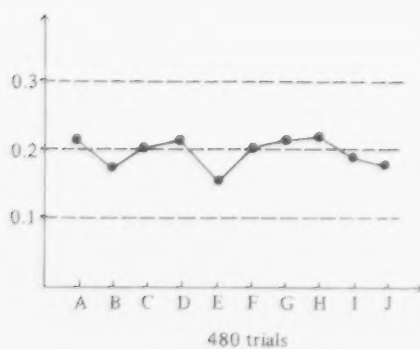
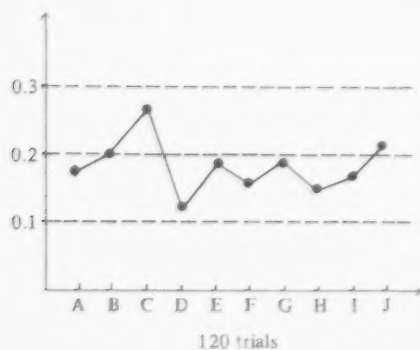


Figure 2

2. Let's challenge an Arabic word test.

The following is an Arabic word test of 50 words. No students know Arabic, so they are forced to choose answers at random.

Therefore, we get samples of Bernoulli trials. Let's compare the results of the experiments with B (50, 1/2).

Test of Arabic words					
(1) أَبٌ	a. father	b. mother	(26) شَيْطَانٌ	a. spirits	b. water
(2) جَبَلٌ	a. mountain	b. sea	(27) صَنْدُقٌ	a. plate	b. spoon
(3) كُرْسِيٌّ	a. desk	b. chair	(28) خُبْزٌ	a. bread	b. butter
(4) كَبِيرٌ	a. large	b. small	(29) نَهْرٌ	a. The River Nile	b. The River Euphrates
(5) فَتًى	a. boy	b. girl	(30) الْيَوْمَ	a. today	b. tomorrow
(6) يَدٌ	a. hand	b. leg	(31) نَظيفٌ	a. clean	b. dirty
(7) عَيْنٌ	a. eye	b. mouth	(32) قَرْيَةٌ	a. town	b. village
(8) سَاعَةٌ	a. clock	b. table	(33) قَمَرٌ	a. moon	b. star
(9) لَحْمٌ	a. meat	b. fish	(34) قَهْلَةٌ	a. coffee	b. fruit
(10) تَفَاحٌ	a. apple	b. orange	(35) صِيْنِيٌّ	a. Chinese	b. Japanese
(11) مَيْدَانٌ	a. field	b. square	(36) رَجُلٌ	a. man	b. woman
(12) حَرْبٌ	a. army	b. war	(37) زَهْرٌ	a. flower	b. letter
(13) صَحْرَاءٌ	a. oil	b. desert	(38) رَسْوَاقٌ	a. tortoise	b. rabbit
(14) سَاحِلَةٌ	a. car	b. airplane	(39) مَعْنَى	a. dictionary	b. meaning
(15) رَبِيعٌ	a. summer	b. spring	(40) سَيِّدٌ	a. people	b. government
(16) شَمْسٌ	a. sun	b. star	(41) كَنْزٌ	a. treasure	b. coconut
(17) أَسْوَدٌ	a. white	b. black	(42) رَافِعٌ	a. captain	b. passenger
(18) فَتْرَةٌ	a. door	b. window	(43) فُجْرٌ	a. lunch	b. breakfast
(19) قَرِيبٌ	a. far	b. near	(44) مَلْحٌ	a. sweet	b. salty
(20) أَسْتَاذٌ	a. professor	b. student	(45) شَمَالٌ	a. north	b. south
(21) أَنَا	a. I	b. you	(46) ائْتَمَعَ	a. Friday	b. Saturday
(22) قَلْعٌ	b. castle	b. king	(47) جَرَى	a. run	b. walk
(23) بَعِيدٌ	a. long	b. deep	(48) بَكَى	a. cry	b. talk
(24) نَعَمْ	a. no	b. yes	(49) تَقَرَّبَ	a. run away	b. come near
(25) نَارٌ	a. wind	b. fire	(50) طَلَحَ	a. snow	b. light

Figure 3

3. Understanding of the concept "Expectation" by simulations

A drunken man returns home from the point S in figure 4. Every time he comes to a forked road, he decides at random which directions (left or right) to go. There are 5 obstacles A, B, C, D and E on the way shown in figure 4, and at each point he must pay so much money as shown below.

- A: a bar where he drinks a bottle of beer
It costs him 3,000 yen.
- B: a ditch into which he falls
It costs him 1,500 yen for cleaning his clothes.
- C: a stone over which he stumbles and he gets injured
It costs him 5,000 yen for the treatment.
- D: a police box in front of which he has a quarrel with the policeman using violence
It costs him 6,000 yen for the fine.
- E: a pub where he drinks a glass of wine
It costs him 500 yen.

Well, how much must he pay before he gets home?

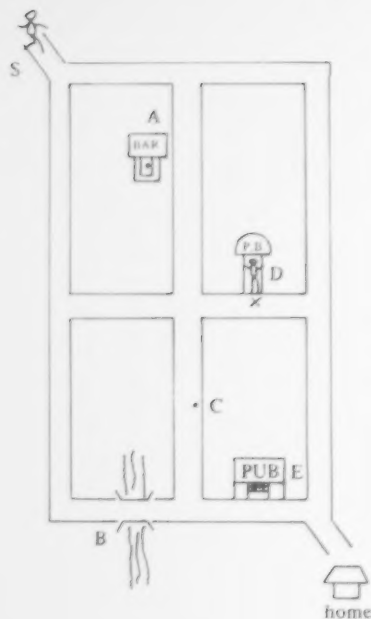


Figure 4

Students can try simulations of his random walks by tossing a coin and collect the data of his cost.

Table 1

trial	1	2	3	4	5	6	7	8	9	10	mean
cost											

How much cost should he expect for a time?

Discussing about this question, students will invent the concept of "Expectation" by themselves.

4. Tape cut

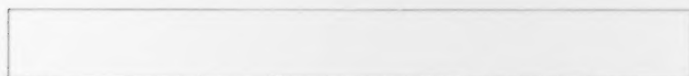
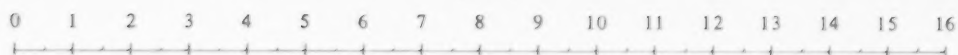


Figure 5

Figure 5 shows the length of 10 cm. Students must learn this length by heart. Then they must cut off 50 pieces of tape each length of which they expect to be 10 cm. Let them measure the true length of each piece and classify them in the table.

Table 2



	rank	frequencies
1	$5.0 \leq x < 5.5$	
2	$5.5 \leq x < 6.0$	
3	$6.0 \leq x < 6.5$	
4	$6.5 \leq x < 7.0$	
5	$7.0 \leq x < 7.5$	
6	$7.5 \leq x < 8.0$	
7	$8.0 \leq x < 8.5$	
8	$8.5 \leq x < 9.0$	
9	$9.0 \leq x < 9.5$	
10	$9.5 \leq x < 10.0$	
11	$10.0 \leq x < 10.5$	
12		
13		
14		
15		
16		
17		
18		
19	$14.0 \leq x < 14.5$	
20	$14.5 \leq x < 15.0$	
21	$15.0 \leq x < 15.5$	

	frequencies														
rank	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1															
2															
3															
4															
5															
6															
7															
8															
9															
10															
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12															
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17															
18															
19															
20															
21															

Students will find that different distributions come out and they understand the effectiveness of the concepts of "the mean" and "the standard deviation" to characterize their works.

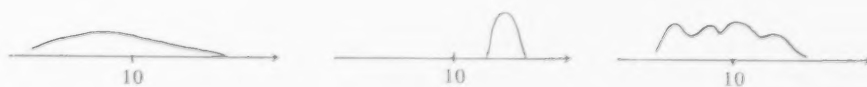


Figure 6

PROBABILITY AND STATISTICS TEACHING IN JAPANESE SENIOR HIGH SCHOOL

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1. Introduction and Summary

The importance of probability and statistics is fully recognized in modern society. Teaching of these things in school is now strongly emphasized all over the world and Japan is no exception.

In this paper, we will examine the present state of probability and statistics teaching in Japanese senior high school mathematics.

We should take note that we are only concerned with probability and statistics teaching within school mathematics and not the whole area, e.g., statistics in other subjects and in other activities of school life.

First we see the present curriculum of probability and statistics in school mathematics focusing on senior high school. Probability and statistics is taught in the subjects, "Probability-Statistics" and "Mathematics II". The major contents are the arrangement of data, probability and its basic laws, probability distribution and the idea of statistical inference. Probability and statistics are not compulsory but rather are optional. Between one quarter and two thirds of pupils study probability and statistics at some time in senior high school mathematics.

Pupils' achievement in the Second International Mathematics Study of International Association for the Evaluation of Educational Achievement (IEA) is good among the twenty participating countries. Pupil's attitudes toward probability and statistics is examined by the data of the same study. Pupils considered solving word problems and proving theorems to be the most important exercises. The second most important was determining probabilities. This was also considered most difficult by pupils. Estimating mean value by sampling survey was not considered so important. Pupils liked this exercise the least of any.

As to the issues in probability and statistics teaching, curriculum is taken up first. Most importantly, not all pupils study probability and statistics in senior high school. Considerable numbers of non-technical course pupils and non-college-bound pupils do not study probability and statistics and have no knowledge and skills beyond junior high school mathematics.

In probability teaching, number of possible outcomes is taught just before studying the concept and basic laws of probability. Often pupils consider determining probability to be only computing a ratio of numbers of possible outcomes, though probability is a measure expressing certainty of indefinite phenomena. This would make pupils misunderstand the concept of probability and give a wrong impression that probability is only a game. Moreover the number of possible outcomes is difficult to compute correctly, and so this may be one of the reasons why pupils considered determining probability most difficult.

In statistics teaching, frequency distribution of two variates and correlation, and time series would be worth introducing into senior high school mathematics. The most essential issue is teaching properties of normal distribution, since statistical inference is based deeply upon them. As mathematical techniques required to deal with these properties are beyond senior high school mathematics, experiments and simulations by microcomputers have been devised. However, these are not necessarily consistent with other mathematics taught in senior high school.

2. Present State of Probability and Statistics Teaching

In the following, we will see the present state of probability and statistics teaching in Japanese senior high school. We will examine two aspects: the curriculum, and pupils' achievement and attitudes.

As mentioned above, we are only concerned with probability and statistics teaching within mathematics classes.

2.1 Curriculum⁽¹⁾

First we will see probability and statistics teaching in the grades preceding senior high school.

In Japan, the initiation of probability and statistics teaching begins in elementary school. An idea of probability is taught informally in the sixth grade and that of statistics of processing data and making tables and diagrams is taught in the third, fourth, fifth and sixth grades.

Subsequent to this, probability and statistics is taught in junior high school.

In the second grade of junior high school (the eighth grade), one of the objectives is to help pupils develop a grasp of mean values and trends of statistical phenomena through using frequency distributions. The contents are to help pupils collect data in accordance with their objectives, and to arrange this data by using tables and graphs, and thereby to ascertain the trends of data by paying to attention to representative values, and variance etc.

In the third grade of junior high school (the ninth grade), one of the objectives is to help pupils understand the meaning of probability and the elementary concepts of a sampling survey, and thereby further deepen their views and ways of statistical thinking. The contents are: (1) to help pupils understand about probability by paying attention to the frequency obtained through large numbers of observations or trials and compute probability in simple cases, (2) to help pupils understand that the trend in a population can be estimated from a sample.

Now we will focus on senior high school mathematics.

In senior high school mathematics, six subjects are established and among of them, "Mathematics I" is required for all students in the first grade of senior high school (the tenth grade), and other subjects, "Mathematics II", "Algebra-Geometry", "Basic Analysis", "Differential and Integral Calculus" and "Probability-Statistics" are optional.

Probability and statistics is taught in the subject "Probability-Statistics" and "Mathematics II". Therefore probability and statistics study is not required for all pupils.

The objectives of "Probability-Statistics" is to strengthen and to help the understanding of

(1) The educational system of Japan is explained briefly in Appendix I, so please refer to it if necessary.

fundamental concepts and laws of probability, the concept of probability distribution, and to develop abilities to think statistically.

The contents of "Probability-Statistics" are (1) Arrangement of data—distribution of the variate, representative value and measure of dispersion—, (2) Numbers of possible outcomes, (3) Probability and its basic laws—addition theorem, independent trials and its probability, conditional probability—, (4) Probability distribution—binomial distribution and normal distribution—, (5) Idea of statistical inference—estimation and test.

The contents concerning probability and statistics in "Mathematics II" are: a. permutation and combination, b. probability and statistics—meaning of probability, probability distribution, an idea of statistical inference—. These are a simplification of the contents of "Probability-Statistics". (Details of objectives and contents are described in the Course of Study, and excerpts concerning probability and statistics are given in Appendix II.)

Probability and statistics is optional, as we mentioned above, how many pupils are studying them in senior high school? We do not have precise information, but from the numbers of textbooks used in mathematics classes we can estimate that the subject "Probability-Statistics" is selected by about 25.3% of pupils at some time during their high school career, and "Mathematics II" is selected by about 40.2%.⁽¹⁾

In "Mathematics II" many contents other than probability and statistics, e.g., vectors, differential and integral calculus, sequences, and computers and flow charts, are listed to be taught and teachers freely choose the suitable contents for their pupils. It is not certain whether teachers teach probability and statistics in "Mathematics II". Therefore, between a quarter and two thirds of pupils study probability and statistics at some time in senior high school.

2.2 Pupils' Achievement and Attitudes

We will see the pupils' achievement and attitudes toward probability and statistics through the data of the Second International Mathematics Study of the International Association for the Evaluation of Educational Achievement (IEA) in Japan.

First we will focus on achievement. The test of the Second International Mathematics Study of IEA was administered to Population B in November 1980. Population B is pupils who are in the normally accepted terminal grade of the secondary education system (in Japan, the third graders of senior high school of full time course), and who are studying mathematics as a substantial part of their academic program.

Following is the summary of the data of probability and statistics:

Table 1. Population B Achievement Mean Scores (Probability & Statistics)

Content area	Score		Opportunity to learn	
	Japan	International	Japan	International
Probability & Statistics	72%	50%	83%	63%
Total	69	—	91	—

(1) Number of tenth graders was 1,829,026 in 1984. In 1985, the number of pupils taking "Probability-Statistics" is 462,610, and that of "Mathematics II" is 732,046 (textbooks sales in 1984 & 1985).

(In Table 1, the scores are the average percentage of correct responses on test items in the content area, and the numbers of opportunity to learn are the average percentage of pupils who learned the materials on the test items already. "International" is the mean value of twenty participating countries.)

The number of test items in the content area "Probability & Statistics" is seven, and they are divided into three behavioral categories—Computation, Comprehension and Application. Numbers of items in each category and their mean scores are as follows:

- * Computation 3 items, 87.5%,
- * Comprehension 1 item, 53.1%,
- * Application 3 items, 62.4%.

Examples of test items in the above categories and scores are shown below:

Outline of Items	Scores
* A set of 24 cards is numbered with the positive integers from 1 to 24. If the cards are shuffled and if only one is selected at random, what is the probability that the number on the card is divisible by 4 or 6? (Computation)	87.2%
* The mean of a population is 5 and its standard deviation is 1. If 10 is added to each element of the population, what is the new mean and standard deviation? (Comprehension)	53.1%
* A test is taken by all first year university students in a country. The mean is 50 and the standard deviation 20. Assuming the scores are normally distributed, approximately what percentage of students score more than 30? (Application)	44.9%

By these data we see that Japanese pupils' achievement and opportunity to learn is high compared with those of other participating countries. One of the reasons why pupils' scores are high is the high rate of opportunity to learn. Their achievement in probability and statistics is almost the same as the average of all content areas.

We should note that pupils are good in computation, e.g., determining probability of simple events, but not very good in comprehension and application, i.e., many pupils are weak in the problems which require high grade ability and full knowledge.

In addition, pupils are rather good in calculation of probability but not so good in probability distribution and statistics. That is to say, many of them could not deal with the probability and frequency distribution, mean value and standard deviation.

Next we will see the pupil's attitudes toward probability and statistics through the data of responses to questionnaires in the Second International Mathematics Study of IEA.

The results of pupils' responses are given in Table 2.

(In Table 2, Response category 3 represents "undecided". The percentage of "No Answer" is 2.2% in each item.)

From these data, we see that among above three items pupils considered determining the probability to be the most important, getting information from statistical tables was second in importance, while estimating mean value by sampling survey was third. Through data not contained in this report, it has been determined that pupils considered solving word problem and proving theorems to be the most important, and determining the probability as number two in importance.

Table 2. Population B Responses to Questionnaires (Probability & Statistics)

Outline of questionnaire	Category of response				
	1	2	3	4	5
* Getting information from statistical tables					
A Importance (1—very important 5—not at all important)	23.8%	43.9%	23.2%	5.9%	1.0%
B Easy (1—very easy 5—very hard)	1.7	13.5	45.9	29.9	6.8
C Like (1—like a lot 5—dislike a lot)	2.9	14.5	45.2	27.3	7.9
* Determining the probability of an outcome					
A Importance	31.7%	46.4%	14.4%	4.1%	1.1%
B Easy	1.5	11.7	34.7	37.9	12.0
C Like	4.7	22.0	35.0	25.9	10.1
* Estimating mean value by sampling survey					
A Importance	18.4%	41.5%	26.1%	9.4%	2.5%
B Easy	1.2	11.6	53.2	25.7	6.1
C Like	1.1	8.8	50.6	28.3	9.0

In Table 2, all three contents were found difficult, and determining the probability was the most difficult.

Most remarkable is the fact that the thing pupils disliked the most was estimating mean value by sampling survey.

3. Issues in Probability and Statistics Teaching

Coming next to the present state of probability and statistics teaching in Japanese senior high school, we will examine the issues under two headings: 1. Curriculum, 2. Contents and Materials.

3.1 Curriculum

As seen in 2.1, in senior high school, probability and statistics is optional, and between a quarter and two thirds of pupils study probability and statistics at some time in senior high school.

In addition, in many high schools in which a large percentage of pupils intend to enter into university, the subject "Probability-Statistics" is selected mostly by the pupils of science courses but not by pupils of non-technical courses. Thus considerable numbers of non-technical course pupils have no knowledge and skills about probability and statistics beyond junior high school mathematics. Non-college-bound pupils mostly choose "Mathematics II", and they may or may not study probability and statistics in the class. Moreover there are pupils who do not

take any optional mathematics. Therefore their situations may be the same as non-technical course pupils, i.e., large numbers of them have only knowledge and skills about probability and statistics of junior high school mathematics.

This fact is serious when we consider the importance of probability and statistics in the coming science and technology society.

3.2 Contents and Materials

First we will examine contents of "Probability-Statistics". They are divided into three parts:

- A. Number of possible outcomes;
- B. Probability and probability distribution;
- C. Statistics and statistical inference.

We point out that A. is fundamental to all mathematics and is not limited to probability and statistics alone. However, to study A. just before B. may have bad influence on probability study. This is because it can produce misunderstanding of the fundamental concept of probability. In fact, determining probability is reduced often to calculating the ratio of numbers of possible outcomes, and pupils assume that probability is only the number obtained by the calculation of numbers of possible outcomes. Actually probability is the measure expressing certainty of indefinite phenomena. As to this point teachers try to help pupils not to fall into faults of misunderstanding. Nevertheless problems and exercises for pupils are chiefly "number of possible outcomes" type. Consequently pupils may well misunderstand true meaning of probability.

Moreover problems of calculating number of possible outcomes are very difficult for pupils to solve, as they are often very tricky and require skillful tactics. No doubt, this is one reason why pupils considered determining probability the most difficult (in Table 2).

Followings are materials from a "Probability-Statistics" textbook—Senior High School Probability-Statistics edited by K. Yoshida, M. Kurita & J. Hashimoto—.

There are ten examples in determining probability, five of which are solved by division of numbers of possible outcomes (Type I), e.g.:

- * If three representatives are selected among six members, A, B, C, D, E and F, then determine the probability of A being one of the representatives.
- * If three balls are taken from a bag which contains seven red balls and five white balls, then determine the probability of two being red and one being white.

Three more examples are applications of basic laws of probability (Type II), e.g.:

- * There are two bags, named A and B. A contains four white balls and six red balls, and B contains five white and five red balls. If a ball is moved from A to B, balls in B are mixed fully and then a ball is moved from B to A, determine the probability of the number of white balls in A being the same as before.

In examples of this type, fundamental probabilities to which basic laws are applied are the same as Type I.

Only one example is related to the statistical probability (Type III), e.g.:

- * Electric bulbs sold in the Town S are produced by one of three companies A, B and C. Market shares of A, B and C in Town S are 45%, 30% and 25%, and percentages of production above the quality standard of A, B and C are 90%, 85% and 87% respectively. If we buy a bulb in Town S, determine the probability of it being above the quality standard.

The last one is to prove the independency of events (Type IV).

In the above textbook, there are twenty five exercises of determining probability. Among of them nine exercises are of Type I, thirteen are of Type II and three are of Type III. We point out that situations dealt with in exercises are mainly confined to coin tossing, die tossing, taking up balls from a bag, and lottery. Although these are very valuable mathematical models for probability, this confinement illustrate why pupils may possibly assume that probability is only for a game.

As to the contents of statistics, it is doubtful that statistics should be limited to frequency distribution of one variate. Frequency distribution of two variates in simple cases and correlation may be appropriate to study in senior high school. Another issue is that time series is not brought into school curriculum. Time series is useful in various areas and mathematical skills to deal with it would be easy within school mathematics. These two points would be worth further discussion.

The most essential part of statistics teaching is an idea of statistical inference—sampling survey, estimation and test.

Since this idea is deeply based upon probability distribution theory, understanding of mathematical laws of fundamental probability distribution, especially those of normal distribution, is indispensable. For studying properties of normal distribution various mathematical techniques are prerequisite. Among of them there are some which are beyond pupils' reach in senior high school mathematics. Typical are an improper integral including exponential function, and a double integration. Therefore, computation of mean and standard deviation of sum of normally distributed independent probability variables is beyond senior high school mathematics.

To compensate for this, experiments or simulations by microcomputers have been devised by several teachers. We know the significance and necessity for devising such means. We have to wonder that the twelfth graders who are in the last stage of senior high school studying mathematics based upon considerably rigid foundations would have a sense of inconsistency. In mathematics class, the pupils learn that they must have proof, but experiments or simulations can only make a guess.

We may assume that one reason why pupils disliked estimating mean values by sampling survey (in Table 2) is this inconsistency. This point may be a serious problem in teaching the idea of statistical inference in senior high school.

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Appendix I Educational system of Japan

Japan's school system is basically a 6-3-3-4 system, i.e., six years for elementary school, three years for junior high school, three years for senior high school (four years for part-time course) and four years for college and university. Other types of schools are added as the needs arise, e.g., two-years junior college, technical college, miscellaneous school and so on.

In this system, elementary and junior high education are compulsory and free, and children of 6-14 years old go to the schools predetermined by the boards of education of their districts, with a few exceptions for those who go to private schools or special schools. The enrollment in compulsory education is 99.99% now.

After graduating from junior high school, students take an entrance examination to senior high school and enter a senior high school on the basis of the results of this examination. According to 1984 statistics, ninety four percent of children of the corresponding age cohort are enrolled in senior high schools.

Looking into the composition of students in terms of curriculum, about seventy percent are in the general education course and thirty percent are in the specialized or vocational education course.

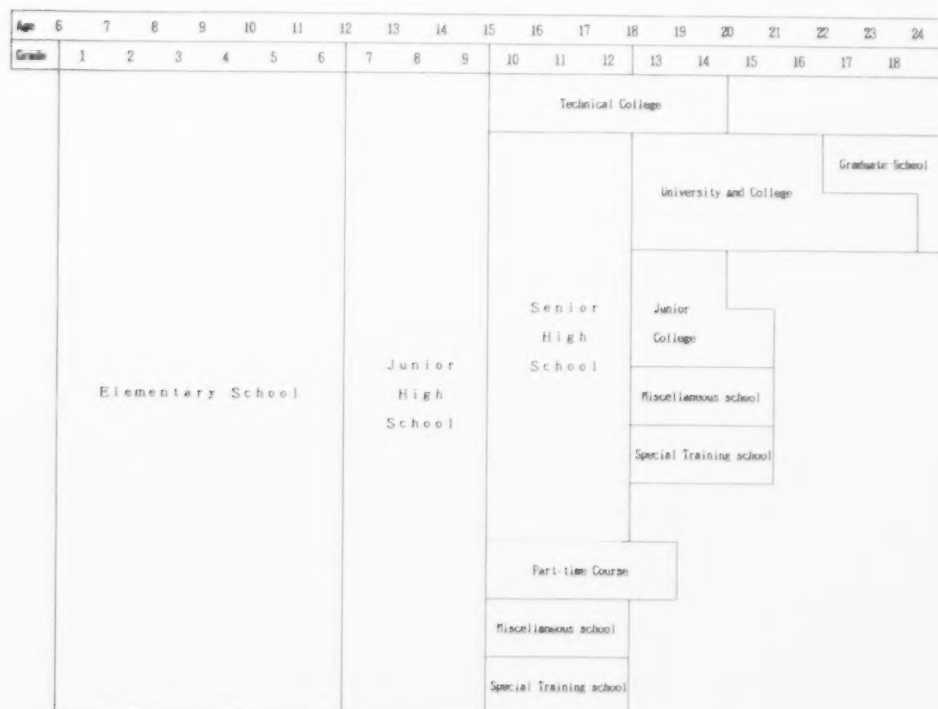


Figure 1. School System of Japan

The enrollment in universities and junior colleges of the corresponding age cohort was thirty five percent in 1983. Selection is done by an entrance examination. The entrance examination of prestigious and famous universities is very difficult, and mathematics is usually one of the key subjects.

The curricula of schools (from elementary school to senior high school) are based upon "The Course of Study", the standard syllabus which is set by the Ministry of Education.

Now in elementary and junior high school, mathematics is compulsory. In senior high school, all tenth graders have to learn "Mathematics I", but in the subsequent grades mathematics is optional. The objectives and contents of the mathematics curriculum are determined in the Course of Study and are the same for all students up to tenth grade, while details and teaching methods are left to school teachers.

Textbooks used in classes are compiled by commercial publishers according to the Course of Study and have to be examined and approved by the Ministry of Education before used in school.

Appendix II Excerpts from the Course of Study — concerning Probability and Statistics —

1. Elementary School

Contents

(A. Numbers and Calculations, D. Quantitative relationship)

* The third grade

D(2) To help children be able to represent data in the simple tables or graphs and to read them.

- a. To classify and arrange data at the simple view points such as day, time and place etc. and to represent them in a table.
- b. To know how to read and how to draw bar-graphs.

* The fourth grade

D(3) To help children develop their abilities to gather, classify and arrange data in accordance with the purposes and to consider their features.

- a. To consider the cases which may happen as regards to the two affairs.
- b. To check up omissions and duplications in data.
- c. To represent the data in bar-graphs or broken-line graphs, and to be able to interpret these graphs.

* The fifth grade

D(4) To help children classify and arrange data in accordance with some objective and to be able to represent data by using circle graphs and tape graphs.

* The sixth grade

A(2) To help children understand numbers and calculations more thoroughly.

- d. To know that numbers are used to represent measure of probability of occurrence of indefinite events.

D(3) To help children develop their abilities of considering and representing statistically more thoroughly, for example, by investigating the distribution of data in simple cases.

- a. To know a frequency table and graphs.
- b. To know that there is a case in which the tendency of the population is conjectured through the ratio gained from a point of statistical data.
- c. To select adequate tables and graphs in accordance with the purposes and to devise some useful ones.

2. Junior High School

* The second grade (the eighth grade)

Objectives

(4) To help pupils develop their abilities to grasp the trends of statistical phenomena through using frequency distributions, mean values, etc.

Contents**D. Probability and Statistics**

(1) To help pupils collect data in accordance with their objectives, and to arrange this data by using tables and graphs, and thereby be able to ascertain the trends of the data by paying attention to representative values, variances, etc.

- a. Meaning of frequency distribution and way of looking at histogram.
- b. Meaning of relative and cumulative frequency.
- c. Meaning of mean value and range.

* The third grade (the ninth grade)

Objectives

(4) To help pupils understand the meaning of probability and the elementary concepts of a sampling survey, and thereby further deepen their views and way of statistical thinking.

Contents**D. Probability and Statistics**

(1) To help pupils understand about the probability by paying attention to the frequency obtained through large numbers of observations or trials.

- a. Meaning of probability.
- b. To compute probability in simple cases.

(2) To help pupils understand that the trend in a population can be estimated from a sample.

- a. Population and sample.
- b. Mean value and ratio in a sample.

Remarks concerning to Contents

(2) As for the content D(1)-b, events which may be easily classified by the aid of tree diagrams, etc. should be dealt with.

(3) As for the content D(2), it should be limited to the level of dealing with experiments and observations.

3. Senior High School*** Probability-Statistics****Objective**

To strengthen the pupils' understanding of fundamental concepts and laws of probability, to help them understand the concept of probability distribution, and to develop their abilities to think statistically.

Contents

(1) Arrangement of data

- a. Distribution of the variate.
- b. Representative values and measures of dispersion.

(2) Number of cases of possibilities

- a. Permutation, combination.
- b. Binomial theorem.

(3) Probability

- a. Probability and its basic laws.
- b. Independent trials and its probability.
- c. Conditional probability.

(4) Probability distribution

- a. Random variables and its probability distribution.
- b. Binomial distribution, normal distribution.

(5) Statistical inference

- a. Population and sample.
- b. Idea of statistical inference.

(Terms & Symbols)

variance, standard deviation, Σ , nPr , nCr , factorial, $n!$, complementary events, exclusiveness, independence, dependence, expectation, estimation, test

Remarks concerning Contents

Content (5) should be limited to gaining an understanding of statistical inference through concrete examples.

*** Mathematics II****Objective**

To help pupils understand fundamental concepts, principles and laws in broader fields of mathematics which follow contents studied in Mathematics I, and thereby to deepen their appreciation of the role played by mathematics in society.

Contents**(1) Probability and Statistics**

- a. Permutation combination.
- b. Probability.
- c. Statistics.

(Terms & Symbols)

nPr , nCr , factorial, $n!$, complementary events, expectation, standard deviation

SOME THOUGHTS ON STATISTICAL EDUCATION

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It is always difficult to specify exactly what one means by "statistical education". In the narrow sense, it probably includes the teaching of statistics in schools, the training of professional statisticians in universities, and perhaps the general understanding of statistical principles by social scientists, and agricultural, industrial or technical research workers.

These educational activities are on the whole well launched, though there exist wide variations between the teaching of statistics in English or Scandinavian schools as against Brazilian or Malaysian schools, for example. A few countries have already established the teaching of statistics in their school curricula, but for a very much larger number this represents a long-term ideal, after literacy and other educational basics have been achieved. Because of the value of statistical thinking, a strong case can be made for the inclusion of statistics among basic educational requirements, together with elementary mathematics at primary and secondary schools.

At the tertiary level, one finds that most universities in developed and in some developing countries have a statistics program. In these, some statistical principles are taught, though there are many institutions where such statistical training remains at a very rudimentary level of development. A similar type of disparity exists in the use of statistical methods for research in agriculture, industry, and technology in developed and developing countries. While, for example, Britain, Australia and the U.S.A. enjoy the advantages of advanced experimental design in their agricultural research, this is decidedly not the case in most African countries. Likewise statistical quality control is used in only a few industrially sophisticated countries, Japan and the U.S.A. foremost among them.

In any general discussion of statistical education, it is important to order one's priorities. It is clearly necessary to advance current academic statistical education, in the narrow sense, by improving the teaching of statistics in schools, universities and research institutes. In this context, the relative weights given to probability and statistics in courses, and the use of calculators and computers in statistical training are issues of relevance. But it is equally important, in a world where numeracy is becoming increasingly essential to economic survival, to ensure that statistical education, in the broader sense also progresses. In other words, it would be desirable for elementary statistical concepts and ideas to form part of the primary and secondary schooling, not only of developed but also of developing countries. This presents us with a formidable problem whose solution will be one of the great professional challenges of the next few decades.

Let me expand a little on what I mean by elementary statistical concepts and ideas. An analysis of *the New York Times* of 22 May 1982 showed that on this particular day, 32.6% or about 1/3 of the newspaper's columns contained statistical material. This included financial

reports, economic problems, social issues, political arithmetic, sporting records and weather information. While none of these categories involved difficult statistics, it was clear that statistical training at least at a basic level (averages, comparison of proportions, graphical statistics, frequency diagrams), was assumed to be an essential part of every reader's education. A more recent study carried out on newspapers in Argentina by R. Mentz has confirmed an even higher statistical content in two newspapers in Buenos Aires and Tucuman.

No citizen unfamiliar with elementary statistics could possibly have understood such news items. It is therefore obvious that popular education in the principles of statistical method must become one of our goals. Even more importantly, since econometrics, the testing of drugs, quality control, and sample surveys (among other activities) rely heavily on statistics, the statistical education of individuals in managerial positions must be one of our main priorities.

The International Statistical Institute has long recognized this problem, and through the three taskforces of its Committee on Statistical Education, has attempted to foster various activities to meet it. Professor Barnett's taskforce has concentrated on the teaching of statistics in schools, while Peter Holmes has edited *Teaching Statistics*, a magazine on statistical ideas for teachers in secondary schools. Professor Loynes's taskforce has dealt with tertiary and technical statistical education, and has been involved in the Statistical Training Program for Africa. Professor Råde's taskforce has been charged with the organization of international conferences and round table meetings on statistical education. The first ICOTS meeting was held in Sheffield in 1982 and the second is due to meet in Victoria, B.C., Canada, in 1986. A round table meeting on the impact of calculators and computers on statistics was held in Canberra, Australia, in August 1984. But despite these numerous activities, the Committee has yet to make serious inroads on the more general problem of popular statistical education.

Faced as we are with this problem, it is important for all of us to determine how our resources, both financial and human, are to be best deployed in advancing statistical education over the next decade or two. My own personal preference would be to maintain our present academic thrust in schools and universities, while greatly expanding general statistical awareness as part of the education of the ordinary citizen. Some statistical concepts and methods, however elementary, could be included in the curricula of primary and secondary schools currently being established in the developing countries. Adult education classes could be started, to provide training in statistical literacy. Television programs could be used as a teaching medium for important statistical ideas. Just as elementary arithmetic is now assumed to be part of everyone's educational background, so also should elementary statistics become a component of every citizen's training.

To achieve this aim will require manpower, financial resources and much hard work. Since statistical education is hardly likely to figure as a priority in anyone's political planning, it will be an unglamorous enterprise. Nevertheless, it is a task that will have to be tackled if the citizens of our world in the year 2000 are to become discriminating users of the statistical material placed before them.

PROGRAMMABLE CALCULATORS TO BE USED IN THE LECTURE OF PROBABILITY AND STATISTICS

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Small programmable calculators are by no means inferior to bigger computers in the sense that the former are equipped with the essential functions of (1) *counting up* and (2) *grouping out* all the possible cases concerned, and (3) *simulating* random events. These are just the minimum request for exercises on the elementary theory of probability and statistics.

Of course, generally speaking, they have only some hundreds of memory units, their operations are slow compared with bigger computers and the display windows show no more than one or a few lines at a time. On the other hand, they can be carried so easily that we can use them everywhere, especially in ordinary lecture rooms, their keyboards are enough simple for beginning students and above all their prices are sufficiently low (today some models among them cost less than ¥10,000).

Here is the reporter's own practice at his university. Every week he carries some dozens of programmable calculators in one or two handy case(s) to his lecture room. The class consists of about 100-200 students. For the first half or some more part of the 90 minute lecture he talks on the theory and related topics from some field in probability and statistics. Then the calculators are distributed among students to practice at the theory just learned. Some of the students are familiar with computer operation, possessing the same or similar pieces of instruments (and such youths are increasing in ratio year after year). The majority of the students, however, are still innocent of the computer treatment, so that some moments have to be devoted to the introduction how to operate or manage the calculators at such and such situations. Experienced students naturally help the beginners. The number of instruments does not match with that of students, which brings cooperation rather than competition among them during the exercise. For instance one piece will be shared among three students. One of them reads the program on the blackboard or in the textbook, another inputs it into the calculator and the process is checked by the third.

After completing the input they perform the exercise one after another, using different initial values to produce different sequences of random numbers. "Radom number key" may be available, but the reporter always asks the students to produce the random numbers by the "arithmetic" way, where each of them shall use one's own registration number divided by 999 as the initial value. This is a device for checking the results afterwards. Thus one beginning week they learn how to generate a long sequence of random numbers, watching what shall be "randomness". Another week they study the existence of various types of probability distributions, generating values which realize some random variable or other. If time permits, the teacher may collect the resulting values from the individuals to make up a stem and leaf diagram on a large sheet of scaled paper.

To illustrate this case, let us successively generate a score of exponential random numbers with a definite parameter. If we regard each value as the time length of "next car's coming", the sum of the values means when the 20th car comes. Each student watches the car flow of her/his own like

---O-----O-----O-O-----O---

moving leftward, finally getting the total time as a realized sum of independent, identically distributed random variables. The class being that large, the stem-and-leaf by these sums is most likely to be normally distributed according to the central limit theorem. This experiment shall be remembered until the class studies on the estimation and on the hypothesis test.

As far as BASIC concerns, each of the programs shall be written within 15 or at most 20 lines of orders. The three elements (1)-(3) mentioned at the beginning are necessary in any program and this length seems sufficient for it.

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HOW TO TEACH LIMIT THEOREMS

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1. Introduction

To meet the various needs of the society, the high school curriculum of mathematics in Japan has changed many times. Especially probability and statistics have recently come to play important roles in the mathematics class. However, as far as their teaching method is concerned, there has hardly been any remarkable improvement, so it seems that we have many problems to study in this field. In this paper, we shall make some remarks on the areas which might cause misunderstanding among the students about the notion of probability.

2. About Distributions

The probability distributions which are taught in Japanese high schools are mainly the binomial and the normal distributions, and other types appear only few times. The former two are certainly very important, however they are often used incorrectly where they should not be applied. Even in the case where they are correctly, there are more convenient distributions for calculating. The next example shows that the Poisson distribution gives a good approximation of the binomial.

3. The Poisson Approximation

In n Bernoulli trials, let the probability of success be p and that of failure be q . If n is large, p is small, and $\lambda = np$ is moderate magnitude, it is convenient to use an approximation to the binomial distribution. We denote $nC_k q^{n-k} p^k$ by $b(k)$, then we have

$$b(0) = q^n = \left(1 - \frac{\lambda}{n}\right)^n,$$

and for large n ,

$$\approx e^{-\lambda}$$

where the sign $\approx$ is used to indicate approximate equality.

Further,

$$\frac{b(k+1)}{b(k)} = \frac{(n-k)p}{(k+1)q} = \frac{\lambda}{k+1} \left(1 - \frac{k}{n}\right) \frac{1}{q}$$

and the right side is nearly as $\frac{\lambda}{k+1}$ for large n . Therefore

$$b(k+1) \approx \frac{\lambda}{k+1} b(k).$$

So

$$b(1) \approx \frac{\lambda}{1} b(0) \approx \frac{\lambda}{1!} e^{-\lambda}$$

$$b(2) \approx \frac{\lambda}{2} b(1) \approx \frac{\lambda^2}{2!} e^{-\lambda}$$

$$b(3) \approx \frac{\lambda}{3} b(2) \approx \frac{\lambda^3}{3!} e^{-\lambda}$$

.....

in general

$$b(k) \approx \frac{\lambda^k}{k!} e^{-\lambda}$$

This relation is called the Poisson approximation to the binomial distribution.

We will show an example for Poisson approximation.

EXAMPLE. We counted the number of the absent students in three classes respectively for 91 days. The following is the statistics in the class A.

Table 1

number of absent students	0	1	2	3	4	total	mean
days	54	29	7	1	0	91	0.505

Certainly that the probability of each individual student's being absent on a fixed day is very small and we can assume any student is approximately in the same condition. So it is proper to apply this example to the Poisson approximation. Then, we get the value of $P(k) = e^{-\lambda} \frac{\lambda^k}{k!}$ for $\lambda = 0.505$ as follows:

$$P(0) = 0.6035, P(1) = 0.1048, P(2) = 0.0769, P(3) = 0.0129, P(4) = 0.0016.$$

So the expected frequencies are as follows:

Table 2

number of absent students	0	1	2	3	4
expected frequencies	54.9	27.7	7.0	1.2	0.1

Compared with the actual data in the table 1, these expected frequencies are very close to them.

In the B and C classes the results were as follows:

Class B

Table 3

number of absent students	0	1	2	3	4
days	39	37	14	3	0
expected frequencies	41.3	32.6	12.9	3.4	0.7

Class C

Table 4

number of absent students	0	1	2	3	4
days	71	18	1	1	0
expected frequencies	70.9	17.8	2.2	0.2	0

Thus, the actual data are well approximated by the Poisson distribution.

4. A Random Walk

The law of large numbers is one of the conceptions which we want high school students to understand. As is well known, the law is as follows:

If S_n is the number of successes in n Bernoulli trials with probability p for success, we have for every $\epsilon > 0$

$$P \left\{ \left| \frac{S_n}{n} - p \right| < \epsilon \right\} \rightarrow 1 \quad (n \rightarrow \infty).$$

The law is very important because the intuitive notion of probability is based on it. But sometimes this conception is misunderstood and used incorrectly. WILLIAM FELLER is warning of these misunderstanding on his book [1] by taking up interesting examples. We will therefore pick up one of them and observe how our students show reactions to it.

The example we take up is a linear symmetric random walk. That is the motion of a particle which starts at the origin and moves a unit length in a unit time on the line to right or left with each probability $1/2$. This motion apply not only to the scientific problems such as the theory of diffusion in physics but also to the many problems in economy or education etc.

We will observe the model of a random walk represented a coin-tossing game. Namely, we image a coin tossing performed at a uniform rate, so that the n -th trial occurs at time n . The particle moves at times $1, 2, \dots$ one unit step to right if the coin lands heads, one unit step to the left if the coin lands tails. Then we represent the motion by a time-space diagram. We shall refer to rectangular coordinates t, x ; for definiteness we imagine the t -axis is horizontal, the x -axis is vertical.

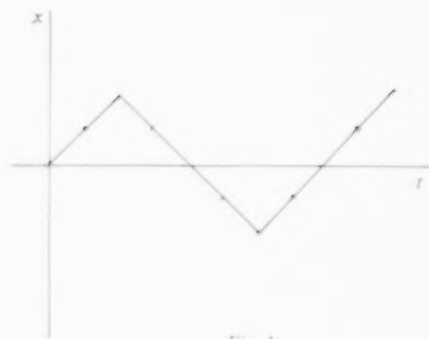


Fig. 1

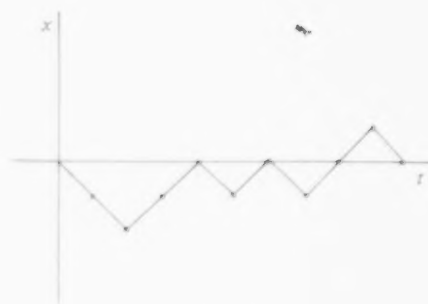


Fig. 2

Suppose that this game is played two men A and B. It is said A is leading while the points on the rectangular line are above the t -axis, in other words, the particle stays on the positive side.

By the case showed in Fig. 1, the particle stays 6 unit-times on the positive side and by the case in Fig. 2, 2 unit-times.

Observing the above, we asked the next question to our students.

QUESTION. Suppose toss over a fair coin 20 times and repeat the trial many times. Which is the most likely case among the next 5, and which is the least probable?

The particle lives in the positive side for the time interval

- (a) $0 \leq t < 2$ (or $18 < t \leq 20$)
- (b) $2 \leq t < 4$ (or $16 < t \leq 18$)
- (c) $4 \leq t < 6$ (or $14 < t \leq 16$)
- (d) $6 \leq t < 8$ (or $12 < t \leq 14$)
- (e) $8 \leq t < 10$ (or $10 < t \leq 12$)

Among the answers to the question

"(e) is the most" 57%

"(a) is the most" 3%

"(e) is the least" 11%

"(a) is the least" 63%

Observing these answers, we see that most of students think of that the case (e) is most likely, that is, the particle spends on the positive side for about a half time interval of 20 unit-times. We will compare their intuition with the result of an experiment.

The experiment is tossing a coin by 20 times and represent their outcomes by a polygonal line. Our pupils enjoyed the experiment, but they were very surprised at the opposite result to what they expected. Indeed, the most probable case is (a) which was least expected by pupils.

On the Table 5, we show the experimental result comparing with the theoretical values which are obtained from the discrete arc sine law.

These values are very close each other.

NOTE. Discrete sine law: The probability that in the time interval from 0 to $2n$ the particle spends $2k$ time units on the positive side and $2n-2k$ time units on the negative side nearly equals as $\frac{1}{n\pi\sqrt{\frac{k}{n}(1-\frac{k}{n})}}$. (See [1]).

Table 5

stays at positive side	0,20	2,18	4,16	6,14	8,12	10
experimental values	0.386	0.179	0.143	0.129	0.087	0.079
theoretical values	0.352	0.186	0.148	0.132	0.124	0.061

5. Summary

We have found an opposite result by the experiment against the students' intuition. We therefore see that such a result comes from their misunderstanding. Tossed a coin many times roughly a half of them land heads. This feeling let them lead obscurely to mistakes. Nothing is said from the theorem about the fluctuation of the lead.

When we observe a sequence of random variables, we are often liable to make mistakes like above. To analyse various informations we appeal to the theories of probability. So we should pay more attention to treating applications of the notion of probability which gives basis to these investigation.

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INDUCING THE POISSON DISTRIBUTION

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Introduction

Today, in the teaching of mathematics at high schools, the stress is put on calculation and demonstration. As a result, students will have only little connection with their daily life in learning mathematics. In order to have well understanding it would be fine if the subjects to be taught have close relation with what they can easily see around.

In this sense, probability and statistics can provide material through which students can consider daily occurrences mathematically. While, only such examples as drawing lots and casting dice are taken up on materials, which students find very boring.

In this reflection, I have collected such examples as students would find familiar in class.

Three Examples

Three examples are now in order.

I. Probability concerning birthday

One of the exciting aspects of probability is unexpectedness. It often happens that results are contrary to what have been expected.

Example 1

What is the probability that at least two students' birthdays are the same in a class of 40 students?

Solution

E: the event that at least two students' birthdays are the same

$\bar{E}$: the event that any two students' birthdays are different

n	20	21	22	23	24	25
$Pr(A)$	0.411	0.444	0.467	0.507	0.538	0.569

The result is unexpectedly that in the class of 23 students the probability is over 50%, in the class of 40, it is 90%.

Students made up this table with electric calculators. Very few expected the result, which made them interested.

Example 2

— Application of Poisson distribution —

A company is going to give the gifts to the children of the employees who were born on the anniversary day of the company. How many articles does the company have to prepare assuming that there are approximately 500 children?

The company is recommended to prepare at least 4. Besides, they learn examples of application of Poisson distribution like this.

II. Probability concerning waiting time for buses

Example 3

Suppose that some passengers are waiting at a bus stop for a bus to come. The buses in the class A start every 15 minutes from 10:00 A.M. till 2:00 P.M. The buses in the class B start every 10 minutes and 20 minutes alternatively also from 10:00 A.M. till 2:00 P.M. Which ones do we have to wait for longer, buses A or B?

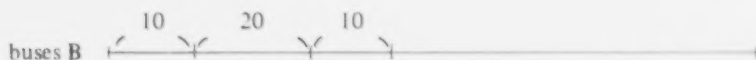
Point: We let students take the continuous quantity as the discrete quantity. Shown in the following the table.



$$P(I_i) = 1/15$$

$X(I)$	14.5	13.5	12.5		0.5
P	1/15	1/15	1/15		1/15

$$E(X_A) = 1/15 (14.5 + 13.5 + 12.5 + \dots + 0.5) = 7.5$$



$$10 \text{ min. } E(X_{10}) = 1/10 (9.5 + 8.5 + \dots + 0.5) = 5.0$$

$$20 \text{ min. } E(X_{20}) = 1/10 (19.5 + 18.5 + \dots + 0.5) = 10.0$$

$$E(X_B) = 5 \cdot 1/3 + 10 \cdot 2/3 = 8.3$$

This problem is the introduction of Poisson distribution. We get this by $i \rightarrow \infty$ of I_i in the table.

On the Gaussian Distribution and Poisson Distribution

Next I should like to explain what I have felt when I showed the students the examples mentioned above.

I. On the Gaussian distribution

As, in most of the textbooks published in Japan, the Gaussian distribution is induced from binomial distribution unnaturally, it is difficult for most of students to understand the true meaning of the Gaussian distribution which is sitting in the center of probability distributions.

Malapplications of the Gaussian distribution are prevailing in the textbooks. Students are able to learn and know its important role which is played in statistics once they know it is practically applied to various social events as well as natural phenomena. My feasible suggestion in this sense is to employ such applications in the class room so as the students can characterize this distribution clearly in spite of little sacrifice of the theoretical aspect of accuracy.

II. Inducing the Poisson distribution

The following is the two advantages.

- (1) The process from the binomial distribution to the Poisson one is easier for students than from the binomial distribution to the Gaussian distribution in taking the limits. When the Gaussian distribution is taken as the limit distribution of binomial one, the central limit theorem is little doubt indispensable. But as the theorem is beyond, they have to use it without proof. On this account they feel the process obscure. On the contrary the process from the binomial distribution to the Poisson one needs exponent, which is familiar to students through the analysis.
- (2) There are many events to which the Poisson's can be applied. They attract students' concern on account of their unexpectedness. It is, however, the reason of the unexpectedness that students get used to the Gaussian distribution.

Reactions of Students

It is popular for students to show problems on birthday or traffic accident as practical examples using the Poisson's. Some students have shown willingness to look for other events as their development.

A: I become familiar with exponent to find

$$\lim_{n \rightarrow \infty} (1 + 1/n)^n = e$$

which was learned in analysis.

B: I feel deeply impressed with the correspondence of the birthday's distribution to that of event which an object falls in a certain narrow strip.

Conclusion

Probability and statistics are, time and again, areas in mathematics for students to be easily familiarized with. This area of study is an optimum one for us to teach them a practical application in mathematics. On the part of students, it is their rewarding opportunity to get over the conventional sense that mathematics is just a training of solving given problems like solving puzzles, and to really appreciate the role which mathematics has been playing in the midst of natural phenomena as the Gaussian distribution plays the central role in the arena of probability. In order to realize this opportunity to the fullest sense of the word, we, mathematics teachers should study a wide range of application of probability and statistics.

PROBABILITY AND STATISTICS IN TEXTBOOKS FOR MIDDLE SCHOOL IN THE PEOPLE'S REPUBLIC OF CHINA

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A new edition mathematical textbook was published in 1979, intended to modernize our education. In this text, there is a chapter of statistics for the junior high school in the last semester (about 12 class hours) and also a chapter of probabilities for high school in the third semester (about 10 class hours).

1. The contents of statistics chapter are the following:

7.1 Population and sample. Two examples—the estimate of the production of corn plant and the radius of explosion of a bomb are given to show that the sampling inspection is necessary in the cases of the population being too large, or the experiment being brokenness, or the cost of the experiment being too expensive. Then the concepts of population and sample are introduced.

7.2 Mean. After introducing the concept of means of population and sample, it is illustrated that the role of sample mean is to estimate the mean of the population, it is also pointed out that if one uses the formulas:

$$\bar{x} = \bar{x}' + a, \quad x = \frac{1}{n} \sum_{i=1}^k x_i f_i,$$

where

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i, \quad x'_i = x_i - a, \quad \bar{x}' = \frac{1}{n} \sum_{i=1}^n x'_i$$

and f_i is the frequency number of x_i in the sample, then the computation of mean will be simplified.

7.3 Variance. An example is exhibited to show that one can distinguish the levels of production of two machines by inspecting the concentration of deviations from the standard quality index of the product, when the means of quality indexes of the product of two machines are equal. Then the concepts of variance and standard deviation are introduced.

7.4 Simplified method for computing variance. The equivalence of

$$(1) \quad S^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2,$$

$$(2) \quad S^2 = \frac{1}{n} \left(\sum_{i=1}^n x_i^2 - n\bar{x}^2 \right)$$

and

$$(3) \quad S^2 = \frac{1}{n} \left(\sum_{i=1}^n x_i^2 - n\bar{x}^2 \right).$$

are checked for the case $n = 3$. And several examples are illustrated to show that the formulas (2) and (3) are simpler than (1) for computing variance.

7.5 Frequency distribution. The concept of frequency distribution, the method for finding the distribution and drawing frequency histogram are introduced.

7.6 Cumulative frequency function.

2. Probability is introduced in the text of high school (chapter 5) after combinations and permutations. The list of content is as follows:

5.1 Probability of random event. The concept of random event, statistical explanation of probability and the property: $0 \leq P(A) \leq 1$ are introduced.

5.2 Probability of equal probable events. Classical model of probability is introduced by using examples. And some elementary examples for finding classical probabilities by combinatorial analysis are also mentioned.

5.3 Probability of the union of mutually exclusive events. The concepts of union events, mutually exclusive events and complementary events are introduced. It is illustrated with example that the finite additivity of probability hold, the formula $P(\bar{A}) = 1 - P(A)$, where $\bar{A}$ is the complement of event A , is proved.

5.4 Probability of the product of independent events. The concept of independent events and formula for probability of independent events occurring simultaneously are studied.

5.5 Repeated independent trials. Bernoulli trials and the general formula for Bernoulli distribution are introduced by examples.

3. Discussion

(i) In our country, we think it may be suitable for the students in junior high school to have some statistical knowledge, for most of the graduates will devote to some occupation or go to occupational high school after they get out of the junior high school. I think the content of statistics in the text is suitable but I think it might be better if the students can do some practical work of statistics under the guidance of teachers.

(ii) After learning knowledge of statistics in junior high school, I think it is suitable to have elementary probability in high school. For it can deepen the cognition of statistics and combinations, and is also very helpful for learning probability and statistics in universities. I think in the high school text the concepts of discrete random variable, its expectation and variance, the

relation to the sample mean and variance should be included. But in virtue of class hours, it can't be accomplished in our textbook.

(iii) In our country, there had been some plans to teach probability and statistics in middle school since late fifties. But since the condition was not possessed, this experiment had not been proceeded until 1978.

Since the present textbooks were published, in some schools these parts of statistics and probabilities have never taught, in some other schools the quality of teaching are not good enough, and there are even doubts about the necessity to teach probability and statistics in middle school. Thus, I believe when we plan to teach probability and statistics in middle school, the most important thing is that the teachers in middle school should believe that this task is necessary and helpful and they also must be well trained and educated in probability and statistics. Now we are happy to see that the course of probability and statistics has been extensively offered in normal universities and teacher's college in our country.

SOME INFORMATION ON SECONDARY SCHOOL TEACHING ON STATISTICS AND PROBABILITY IN USA

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§1. Teaching of Statistics and Probability in American Secondary Schools

In the Bulletin of the National Center for Education Statistics, May 1984 [3], the following results were reported from an analysis of about 12,000 high school transcripts of 1982 graduates: 63.1% of the students took Algebra I, 48.2% took geometry, 31.2% took Algebra II, 7.4% took Trigonometry, 13.4% took other advanced math., 5.6% took calculus, and only 1.2% took statistics.

In 1959 the Commission on Mathematics of the College Entrance Examination Board recommended a one semester course, Introductory Probability with Statistical Applications, as one option for the 12th graders [1]. The course has both practical and mathematical appeal. Probability and statistics are very useful for almost everyone since most people have to make decisions based on essentially statistical data. However, the course has not been as popular as hoped. Calculus is a strong competitor in the curriculum. High school teachers feel more competent to teach calculus than statistics.

In 1983, the Madison Commission on Excellence in Education reported that USA is a "Nation at Risk" [2]. The report indicated that the students in USA are insufficiently trained in mathematics, science and related areas. One of the five major points the report makes about the teaching of mathematics is that high school students must be equipped to understand probability and statistics. Several organizations, e.g. National Council of Teachers of Mathematics, National Council of Supervisors of Mathematics, Conference Board of the Mathematical Sciences, Priorities in School Mathematics, and American Association for the Advancement of Science, expressed their concern about teaching of probability and statistics in the secondary schools. In particular, the Conference Board of the Mathematical Sciences recommended in 1982 that statistics and probability be regarded as fundamental and that appropriate topics and techniques from statistics and probability be introduced into the curriculum [2].

§2. Topics for Statistics and Probability in Secondary Schools

For the junior high school, there is not a separate course in statistics. However, the general mathematics course for these grades should contain such topics from descriptive statistics as the average (mean, median, and mode), frequency distributions, histograms, measures of dispersion, standard deviation, the graphic interpretation of data, and the making of graphs. The mathematical computation required for junior high school statistics is not very difficult. The em-

phasis should be in the understanding of the concepts and the ability to read and interpret the statistical charts.

For the senior high school, probability and statistical inference should be offered as a course in the twelfth grade. The treatment will be more sophisticated since the students have already finished at least algebra. The fundamental concepts such as outcomes, sample spaces, events, probabilities (based on the experience or theoretical consideration) must be emphasized. Topics for the course should include counting, permutation and combination, binomial and normal distributions, central limit theorem, statistical inference such as opinion polls, correlation and its interpretation.

The microcomputers are becoming more available in the secondary classrooms throughout the country in USA. They can be used by the students to handle large real data sets, to simulate the probabilities of complex events, to plot the graphs, etc. The guidelines for computer uses and computer software are being developed under a project "A Program to Improve Quantitative Literacy in the Schools" by Professor Richard L. Scheaffer of the University of Florida [2]. A model in service program and a set of curriculum materials for teachers, among other things, are to be produced under this project.

§3. Teacher Training Programs in Statistics

In the summer of 1985 there was a series of week-long institutes on statistics for high school mathematics teachers. The main focus of each institute is on statistics in society. The topics include exploratory data analysis, sampling and experimental design, probability and simulation, statistical inference, and problem solving as it applies to statistics. One area of concentration is on ways to incorporate statistics into the secondary school curriculum. The institute sites are George Mason University (Fairfax, Va.), State University of New York (Purchase, N. Y.), Grand Valley State College (Allendale, Mich.), Boston Science Museum (Boston, Mass.), and Trinity University (San Antonio, Texas).

References

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- [2] Scheaffer, R. L.: Statistics and probability in the school mathematics curriculum, A review of the ASA/NCTM/NSF Quantitative Literacy Program, 1985.
- [3] The Statistics Teacher Network, Issue #9, April, 1985.

PROBABILITY AND MATHEMATICS IN THE CURRICULUM OF FRENCH TERMINAL HIGH-SCHOOL CLASSES AND UNIVERSITY FRESHMEN AND SOPHOMORES

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I. INTRODUCTION

The importance of mathematics in the curriculum of french students is well known. The reasons for this are not totally glorious. Part of it is tied with an unpenitent need for total ordering and selection. From the beginning of the nineteenth century the ability for the abstract mathematical game has been a privileged parameter in the ranking of people. The whole educational system is polarized by the existence of elitist schools which select their students mainly on the basis of mathematics, after two years of intensive preparation following high school.

The program of those "preparatory classes" ("classes préparatoires": located in about 135 public institutions around the country) is fixed nationally by the ministry of education, which also fixes by "decrets" the programs of classes in high-schools (public or private!). It is sometimes hard for foreigners to totally figure out the somehow "frightening" coherence of this system.

For the majority of freshmen and sophomore students (those who are not among the five thousand admitted every year in preparatory classes) the universities offer a slightly less rigid system. For 15 years universities indeed have been free to determine their programs. But the behavioral unity of the french community is such that it does not make much difference.

In this communication we do not intend to develop long considerations. After noting a few features of the studies, we shall present selected documents on programs, with a few comments. These problems are typical of what is expected from students after standard scientific studies at the freshman and sophomore stage in "preparatory classes".

II. SOME FEATURES OF THE STUDIES

1. The disappearance and shy reappearance of geometry

Geometry and even geometric drawing played once a privileged role in the mathematical education. Geometry has completely disappeared in "non scientific" sections of high-schools where the program includes in contrast the study of primitives, of functions log and exp and some linear algebra. Those subjects seem to be considered as "less abstract than geometry!" in scientific sections some geometry is present as can be seen from programs given in the documents below.

2. The dominant place of Analysis

The student has an introduction to basic calculus during the two terminal years at high school.

Some abstract dealing with convergences and limits is also part of the high school program. Linear algebra (with a formal definition of vector spaces) is also present in all classes of senior high school.

3. The almost total absence of probability in the curriculum of scientific sections in high schools and in the "preparatory classes"

In high school there is almost no probability in the official programs outside some elementary combinatorial problems. Some descriptive statistics are introduced which busy teachers and students for a couple of hours in the years.

In the preparatory classes⁽¹⁾ (Freshmen and sophomores), except those which are intended to orient students toward studies in biology or agriculture, there is no probability and no statistics, as can be seen from the documents below. In the courses for biologists probabilities are excuses either for combinatorials or multiple integrals.

In universities Freshmen and sophomores are in general offered an optional annual course in probability. This course is usually not measure-theoretic even though in many cases it uses some vocabulary of measure theory. It deals separately with discrete and continuous random variables. In most cases it is a good introduction to probability theory which is usually presented in a measure theoretic treatment during the third year of mathematical studies at University.

4. The time devoted to mathematical teaching

The time devoted to mathematics in the last year of high school ranges from 2 hours a week (in "literary" sections) to 8 hours (in scientific sections "C" and "E"), on a total of 26 hours of teaching a week.

In "preparatory classes" students receive weekly 14 hours of teaching in mathematics. We give below an abstract of the program imposed by the Ministry of Education for those classes.

III. DOCUMENTS

1. Official program of a scientific section in the terminal year of high-school

We give now the detailed program as imposed to all high-schools by the Ministry of Education, for the terminal year in the scientific sections called "C" or "E". The reader will judge by himself how negligible is the place of probabilities and statistics. He also will observe to which extent the program is detailed and allows only an extremely small amount of initiative, not only in the choice of topics but also in the presentation. The official text of the program is introduced in the following way:

"In the preceding classes the students have acquired an important scientific knowledge which will be used from the beginning of the terminal year in many different ways. The teacher should remember that all materials (admitted or proved) in the previous classes are supposed to be available.

In the text of the program, the expressions 'admitted statement' or 'one will admit' indicate a proposition for which the opportunity of the proof will be left to the judgement of the teacher. The knowledge of such a proof will not be required at the 'baccalauréat-

(1) See Note (2) for a description of the place of those "preparatory classes" in the french system of education.

examination⁽²⁾. All the other propositions will be proved. In previous classes the students have learned about some general mathematical structures. These ones should not be extensively studied for themselves: but it is important to have them available and to use the appropriate language in studying, for example, complex numbers or the exponential function. The teacher will therefore introduce at the appropriate time the notion of a 'commutative field without producing other examples than $\mathbf{R}$, $\mathbf{Q}$ or $\mathbf{C}$, groups, subgroups, morphisms of groups...'

A. COMBINATORICS – STATISTICS

a) Number of mappings from a finite set in another finite set. Number of injections. Arrangements. Number of subsets of given cardinal of a finite set; Combinations.

Notations: $C_n^p, \binom{n}{p}$

Formulas: $C_n^p = C_n^{n-p}, C_{n+1}^{p+1} = C_n^p + C_n^{p+1}$

One will treat many examples, showing the relation with the calculation of probabilities in the case of the equiprobability of discrete events.

b) Binomial formula.

c) Simultaneous study of two real valued variables measured on a population of m individuals, the associated cloud of points in $\mathbf{R}^2$.

Fitting an affine function to m experimental points by the method of least squares. Regression lines. Coefficient of linear correlation. Case of weighted points. Barycenter of a cloud inertia of a cloud around a point. Minimum of this inertia.

B. NUMERICAL SEQUENCES

a) Fundamental property (should be admitted): every increasing, bounded from above sequence (resp. decreasing bounded from below) converges⁽³⁾.

b) Sequences diverging towards $+\infty$ (resp. $-\infty$). Stability of the behaviour of such sequences when adding a bounded sequence or when multiplying by a sequence admitting a strictly positive lower bound (can be admitted). Study of the sequences: $n \rightarrow a^n$ and $n \rightarrow n^\alpha$. Comparison of their increase.

(2) This is the examination at the end of high-school. The success to this examination (containing mathematics, physics, chemistry, literature, biology, foreign languages, history, geography, philosophy and some optional topics like sport, music, drawing, etc.) gives the student the right to be admitted in the university of his "circumscription" (the circumscription depends from his living place). If the student wants to enter "preparatory classes" (institutions parallel to universities: sort of colleges specializing in the preparation of entrance examinations in "Grandes Ecoles") in order to compete, two years later, for entering some "Grande Ecole" (at the "top" of which Ecole Normale Supérieure and Ecole Polytechnique), he has to get extremely good marks at the "baccalaureat" and to be able to present good continuous records from his high-school. Every institution (most of them public, with no tuitions, very few private) is entirely free to accept whom it wants, regardless of the students' circumscription. Some of them, like "Lycée Louis Le Grand" in Paris, which has a tradition of a high rate of success in the entrance examinations, draw a big concentration of top-students. Naturally this helps them keeping their tradition: it is a very stable process! Among the 50,000 students who get the baccalaureat C or E, only about 8,000 are admitted in some preparatory class. Almost all of them (after two years or sometimes three when they double their attempt) will enter some school: if not one of the most famous "grande école" at least some engineering school.

(3) Comment: the rigorous definition of convergence has been given in previous years.

c) Recurrent sequences.

Examples of sequences such that $u_{n+1} = f(u_n)$.

Examples of study of sequences such that $u_{n+1} = au_n + bu_{n-1}$.

where a and b are given real numbers or $u_{n+1} - u_n = P(n)$.

where P is a polynomial. Some of these examples will be taken from the description of some evolution models in economics or biology.

C. NUMERICAL FUNCTIONS

a) Neper-logarithm. Function $x \rightarrow \ln x$ should be presented as soon as possible using what students know from previous years. The function $x \rightarrow \log_{10} x$ will be introduced in order to be used for numerical calculus.

b) Complements on continuity and limits. Composition of a function having a limit ℓ at point x with a function continuous at point ℓ .

If a function is increasing on an interval $]a, b[$ ($a < b$), and if it is bounded, then it has a limit at b (admitted).

Functions tending to $+\infty$ (resp. $-\infty$). Stability of this asymptotic behaviour when adding a bounded function or multiplying by a positive function bounded away from zero (admitted).

c) The following three fundamental properties of continuous functions on an interval (closed or not) will be presented and explained without proof.

The continuous image of an interval is an interval;

The continuous image of a closed interval is a closed interval;

A continuous strictly monotone mapping from an interval onto an interval admits a reciprocal function which is continuous and strictly monotone.

d) Complements on the calculus of derivatives: chain rule, derivative of a reciprocal function; case of $x \rightarrow \sqrt{x}$. Derivatives of higher order.

The notations

$$\frac{df}{dx}, \frac{d^2f}{dx^2}, \dots$$

will be indicated, but the notion of differential is not in the program.

For the needs of physical sciences, the definition and notation of partial derivatives of a function of two or three real variables will be given.

The study of functions using their derivatives stated in the previous years will be pursued on numerous examples: variation, sign, extrema, behaviour at infinity ("asymptotic curves": $y = f(x)$ and $y = g(x)$ when $\lim_{x \rightarrow +\infty} f(x) - g(x) = 0$ and $\lim_{x \rightarrow -\infty} f(x) - g(x) = 0$).

Application to the solution of equations and inequations.

e) Exponential function (it is advised to study this function right after speaking of reciprocal functions). Notations e^x, u^v . Functions $x \rightarrow a^x$ and $x \rightarrow x^\alpha$.

Comparison of the rate of increase of $x \rightarrow \ln x$, $x \rightarrow x^\alpha$ and $x \rightarrow \exp x$.

One will prove, for $\alpha > 0$; the following properties:

$$\lim_{x \rightarrow +\infty} \frac{\ln x}{x^\alpha} = 0, \quad \lim_{x \rightarrow 0} x^\alpha, \quad \ln x = 0$$

$$\lim_{x \rightarrow +\infty} \frac{x^\alpha}{\exp x} = 0, \quad \lim_{x \rightarrow -\infty} |x|^\alpha \cdot \exp x = 0$$

Application of the chain rule to functions of the form $\ln f, \exp f, f^\alpha$.

f) Examples of limited expansions in the neighbourhood of 0: the teacher will restrict himself to give the definition of a limited expansions and will establish the limited expansion up to their third non zero term of $x \rightarrow \cos x, x \rightarrow \sin x, x \rightarrow \exp x, x \rightarrow \ln(1+x), x \rightarrow \sqrt{1+x}$.

Use of limited expansions to study limits. The systematic study of the operations on limited expansions is out of the scope of the program.

g) Statement without proof of Rolle-theorem; geometrical interpretation.

If f is a continuous function of $[a, b]$ with derivative on $]a, b[$, then: If the derivative function f' takes its values between m and M then

$$m \leq \frac{f(b) - f(a)}{b - a} \leq M;$$

if f' has a limit ℓ at point a , then

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = \ell.$$

Extension to the case of an infinite limit.

D. INTEGRAL CALCULUS

a) Integral of a continuous function: it is recommended to take the following definition: let f be a continuous mapping from an interval $I \subset \mathbf{R}$ into $\mathbf{R}$. One has admitted in the previous year that f admits primitives on I and that the difference of two primitives is a constant. As a consequence, for every $(a, b) \in I^2$ the real $F(b) - F(a)$ is independent from the chosen primitive F of f ; it is denoted by $\int_a^b f(t)dt$ and called integral from a to b of the continuous function f . In other terms $x \rightarrow \int_a^x f(t)dt$ is the unique primitive of f on I which takes the value 0 at point a .

The following properties have to be studied: additivity with respect to intervals, positivity of the integral: if $a < b$ and $f \geq 0$ then $\int_a^b f(x)dx \geq 0$; mean value of the integral, affine change of variable; integration by part.

One will study some examples of functions $x \rightarrow \int_a^x f(t)dt$ where f has no explicit primitive.

b) Approximation of an integral: one will present only the method of rectangles with majoration of the rest and will deduce an interpretation of the mean value of a function as limit of a sequence.

c) Application of the integral calculus to the evaluation of the area of the subset of the plane defined by

$$\left\{ \begin{array}{l} a \leq x \leq b \\ 0 \leq y \leq f(x) \end{array} \right. \quad \text{where } f \text{ is a positive continuous function on } [a, b]$$

d) Without general theory other applications of integral calculus will be given: in geometry, mechanics, physics; examples of computation of a volume, a mass, an inertia-momentum.

e) Resolution of linear homogeneous differential equations of first and second order with constant coefficients. In each case the existence and uniqueness of the solution for given initial conditions.

Numerical examples of resolution of equations of the type $y'' + hy' + ky = a \cos(\omega x - \phi)$.

E. VECTOR FUNCTIONS AND KINEMATICS

a) Vector function of a real variable with values in R^2 or R^3 or C (identified to R^2). Definitions and proofs will be given with the help of coordinates.

Derivative of a vector function. Derivative of a sum, of a product $(\phi \vec{V})$ (where $\vec{V}$ is vector valued and ϕ scalar valued), of a scalar product of a vector product. Derivative of the norm of a vector function.

b) Simple examples of parametrically defined curves. (Any study of singular points or infinite branches is beyond the scope of the program).

c) Movement of a point: trajectory, velocity, acceleration. Accelerated and slowed down movements.

Movements on a line, a circle: uniform movement on a circle, harmonic oscillator (on the line).

F. COMPLEX NUMBERS

a) The field of complex numbers: no particular method of presentation is imposed. Bijection $(a, b) \rightarrow a + ib$ from R^2 onto C .

Geometric representation of complex numbers. Conjugate complex numbers. Modulus; triangle inequality; modulus of a product.

Complex numbers with modulus 1; argument of a non zero complex number; notation

$$re^{i\phi}; \text{ relation } e^{i\phi} e^{i\phi'} = e^{i(\phi + \phi')}.$$

Derivative of $t \rightarrow e^{it}$.

b) Complements of trigonometry: De Moivre Formula. Transformation of products into sums and of sums into products. Reduction of $a \cos x + b \sin x$ (setting $a = r \cos \phi$, $b = r \sin \phi$).

c) n^{th} roots of a complex number; group of the n^{th} roots of 1; geometrical interpretation.

Complex solutions of an algebraic equation of degree 2.

G. LINEAR ALGEBRA

The definitions of a vector space and a linear mapping have been given in the previous years. The notion of subspace will be added. This notions will be used in various examples of finite dimensional vector spaces starting from the example of R^n .

a) Operations in R^n . Canonical basis starting from the study of linear combinations of elements in R^n the notion of subspaces generated by a family of vectors will be introduced. Representation of a linear mapping by a matrix.

Determination of a linear mapping from R^p into R^n by the images of the elements of the canonical basis.

The composition of two linear mapping is a linear mapping.

b) Interpretation of a system of n linear equations with p unknown. Decomposition of a vector on a given system of vectors.

c) Gauss-pivot-method (research of a triangular form for a matrix). Its use to determine whether a finite family of vectors is free, if it generates the space.

From this study one deduces the fundamental following results: every basis in $\mathbb{R}^n$ has exactly n elements. Every free family in $\mathbb{R}^n$ has at most n elements and it is a basis *iff* it has n elements. Every generating family in $\mathbb{R}^n$ has at least n elements and it is a basis *iff* it has n elements.

Theorem of the incomplete basis.

d) Solution of a system of linear equations using operations on the lines.

e) Finite dimensional vector spaces: basis; isomorphism with $\mathbb{R}^n$ of a vector space with a basis of n vectors. Dimension. An injective (resp. surjective) endomorphism of a finite dimensional vector space is one to one.

Supplementary subspaces; projections; symmetries.

H. GEOMETRY

a) Given n weighted points (A_i, α_i) study of the functions

$$M \rightarrow \sum_{i=1}^n \alpha_i \vec{MA}_i \text{ and } M \rightarrow \sum_{i=1}^n \alpha_i \|\vec{MA}_i\|^2.$$

Affine mappings: definition, characterization as having the property of keeping invariant barycenters. Image of a line, a plane, of a convex set by an affine mapping. Image of two parallel lines.

It will be shown that the isometries are affine mappings for which the scalar product is invariant. The student should be familiar with translation, homotheties, symmetries (orthogonal or not), affinities: they will use these transformations in numerous problems of geometric constructions and of identification of sets of points having a given property.

b) In the oriented plane: measure of the oriented angle of two lines. Condition for four points to be on the same circle.

c) Plane geometry: group of displacements. Composition of a displacement and an orthogonal symmetry. Skew displacements. Examples of identification of the subgroup of isometries leaving invariant a given subset. The group of similitudes.

Examples of transformations defined by an application $z \rightarrow f(z)$ from $\mathbb{C}$ into $\mathbb{C}$, case of direct similitudes; examples of affine transformations.

d) Spatial geometry

Isometries leaving invariant a given point. The group of rotations with given axis; it will be proved that a rotation can be expressed as the composition of two orthogonal symmetries with respect to two planes.

Composition of two rotations around axes in the same plane; composition of a rotation of axis D and a translation leaving D invariant.

e) Plane geometry

Conics: definitions with the focuses, and with the focus and the associated directing line. Equations. Equivalence of the various definitions. Equation of the hyperbola with the asymptotes as coordinate-axis.

Examples of parametric representation of a conic.

Tangents to a conic in one point (bisectrix property with respect to the focuses).

2. Abstract of the program of "preparatory classes"

We give only the titles of this program (the details as given by the Ministry of Education would occupy more than 15 printed pages!). The reader will see that no probability and no statistics are part of this program. It is thus usual that a scientific student reaches the level of the third university year with no other knowledge of probability and statistics than what is contained in the title "Combinatorics statistics" given above as part of the program in mathematics of the terminal high-school-year.

The weakly teaching in mathematics in preparatory classes is 14 hours.

A. ALGEBRA

Groups. Polynomials and rational fractions. Linear and multilinear algebra. Matrix calculus. Diagonalization of matrices, etc. Quadratic forms. Euclidean and hermitian spaces.

B. ANALYSIS

- Sequences and series.
- Metric spaces. Compact sets. Complete metric spaces. Series in a normed vector spaces. Banach spaces. Prehilbert and Hilbert spaces... Functions of a real variable. Derivatives and differentials. Taylor-formula. Riemann-integrals. Improper integrals.
- Series of functions. Fourier series. Differential equations. Functions of several variables: differentials and derivatives, primitive of a differential form of degree 1.
- Geometry: curves (local study, curvature, torsion...). Local immersions in $\mathbf{R}^n$. Curves and surfaces...

IV. CONCLUSION

I cannot keep from thinking that in the french attitude towards knowledge, the study of statistics is somewhat "vulgar" compared with "high-spirited" mathematics. They are present in the programs only when unavoidable and for this reason the time devoted to them in High-school is slightly more important in sections oriented towards economics or biology.

It is also interesting to note that the students who, after the preparating classes (program above), will enter "Ecole Nationale Supérieure de la Statistique et des Etudes Economiques (E.N.S.A.E.) which educates most of french specialists in the field, will have learned no more statistics than described above before entering the school (at the age of 20). The entrance examinations is based on several written (4 hours) problems in topology, algebra and numerical calculus, literature, foreign language, plus oral examination. But no trace of probability and statistics.

It is possible that the increasing use of computers in the high-schools will create the conditions for a change of attitude towards statistics and make clear the necessity of devoting more time to think about datas, their organization, their presentation, their contingency, the "information" they carry and how to make decisions from this information. But it is certainly too early to observe the least visible sign of such a move.

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